"PHLYZICS"

Kepler's Laws



- 1st Law: The planets move about the sun in ELLIPTICAL orbits, with the sun at one focus of the ellipse.
- 2^{nd} Law: The straight line joining the sun and a given planet sweeps

 \rightarrow Can be remembered as _____

- 3rd Law: The square of the period of revolution of a planet <u>ABOUT THE SUN</u> is proportional to the cube of its mean distance from the sun.
 - → Stated in equation form as _____

Defining the variables:

T =
R =
r =

Some key things to remember/know about Kepler's Laws

1st Law:

- ✓ Circles have centers. Ellipses are like flattened circles, that don't have a center, but rather have two _____.
- ✓ <u>Eccentricity</u> may be interpreted as a measure of how much an orbit's shape deviates from a circle.

For a circle, e = 0For an ellipse, 0 < e < 1 (the lower the e value, the more circular the orbit)



✓ e = 0.017 for Earth's orbit, e = 0.093 for Mars' orbit, e = .252 for Pluto's orbit

2nd Law:

- ✓ Planets move _____ when they are on the side of their elliptical orbit that is closest to the sun.
- ✓ Between March 21 and September 21, there are three days more than between September 21 and March 21. These two dates are the spring and fall equinoxes, when the days and nights are of equal length. Between the equinoxes, the Earth moves 180° around its orbit with respect to the sun. Using Kepler's 2nd Law, explain clearly how you can determine the part of the year during which the Earth is closer to the sun.

3rd Law:

- ✓ All planets that orbit the sun have the same Kepler Constant (which equals ______)
- ✓ All "things" (little) that orbit the same "THING" (BIG) have the same Kepler constant.
- \checkmark The orbital period of the earth about the sun is approximately _____.
- \checkmark The orbital period of the moon about the earth is approximately _____.
- \checkmark When using Kepler's 3rd Law, make sure to use units of METERS and SECONDS.
- \checkmark The units of K are _____.
- ✓ When using your calculator with BIG numbers that involve exponents, make sure to utilize parenthesis properly, making sure to pay attention to the ORDER OF OPERATIONS (remember PEMDAS)
- ✓ If you solve Kepler's 3rd Law for R, it will involve a cube root. There are two ways to do this on your calculator.

Object	Mass (kg)	Radius of object (m)	Period of rotation on axis (s)	Mean radius of orbit (m)	Period of revolution of orbit (s)	Kepler constant R^3/T^2 (n^3/s^2)
Sun	1.98 x 10 ³⁰	6.95 x 10 ⁸	2.14×10^6			
Mercury	$3.28 \ge 10^{23}$	2.57 x 10 ⁶	5.05 x 10 ⁶	5.79 x 10 ¹⁰	7.60 x 10 ⁶	
Venus	$4.83 \ge 10^{24}$	6.31 x 10 ⁶	2.1×10^7	$1.08 \ge 10^{11}$	$1.94 \ge 10^7$	
Earth	5.98 x 10 ²⁴	6.38 x 10 ⁶	8.61 x 10 ⁴	1.49 x 10 ¹¹	3.16×10^7	
Mars	$6.37 \ge 10^{23}$	3.43 x 10 ⁶	8.85×10^4	$2.28 \ge 10^{11}$	5.94 x 10 ⁷	
Jupiter	1.90 x 10 ²⁷	$7.18 \ge 10^7$	3.54×10^4	$7.78 \ge 10^{11}$	$3.74 \ge 10^8$	
Saturn	5.67 x 10 ²⁶	6.03×10^7	3.60×10^4	$1.43 \ge 10^{12}$	9.30 x 10 ⁸	
Uranus	$8.80 \ge 10^{25}$	2.67×10^7	3.88×10^4	2.87 x 10 ¹²	2.66 x 10 ⁹	
Neptune	1.03 x 10 ²⁶	2.48×10^7	5.69 x 10 ⁶	$4.50 \ge 10^{12}$	5.20 x 10 ⁹	
Pluto	$6 \ge 10^{23}$	$3 \ge 10^6$	5.51 x 10 ⁵	5.9 x 10 ¹²	7.82 x 10 ⁹	
moon	7.34 x 10 ²²	$1.74 \ge 10^6$	2.36×10^6	3.8×10^8	2.36×10^6	

1. Using the table below, find the Kepler Constant for each of the objects below (including the moon, but excluding the sun). Explain why the answers make sense.

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- 2. A planet's mean distance from the sun is  $2.0 \times 10^{11}$  m. What is its orbital period?
- 3. If a small planet were discovered whose orbital period was twice that of the Earth, how many times farther from the sun would this planet be?
- 4. Using the data from the table below, determine the Kepler constant for any satellite of the Earth. (Note: The moon is a satellite of the Earth.)

| Planet  | Mass $(m_E=1)$ | Satellite | Orbital Radius<br>(x 10 <sup>6</sup> m) | Period (d) |
|---------|----------------|-----------|-----------------------------------------|------------|
| Earth   | 1.00           | Moon      | 384.4                                   | 27.322     |
| Mars    | 0.107          | Phobos    | 9.38                                    | 0.319      |
|         |                | Deimos    | 23.46                                   | 1.262      |
| Jupiter | 318            | Thebe     | 221.9                                   | 0.675      |
| -       |                | Io        | 421.6                                   | 1.769      |
|         |                | Europa    | 670.9                                   | 3.551      |
|         |                | Elara     | 11,737                                  | 259.7      |
| Saturn  | 95.2           | Janis     | 151.47                                  | 0.695      |
|         |                | Mimas     | 185.54                                  | 0.942      |
|         |                | Calypso   | 294.67                                  | 1.888      |
| Uranus  | 14.6           | Miranda   | 129.4                                   | 1.414      |
|         |                | Ariel     | 191.0                                   | 2.520      |
|         |                | Oberon    | 583.5                                   | 13.463     |
| Neptune | 16.72          | Triton    | 355.3                                   | 5.877      |
| -       |                | Nereid    | 5,510                                   | 360.21     |
| Pluto   | 0.002          | Charon    | 19.7                                    | 6.387      |

### Natural Satellites in the Solar System<sup>\*</sup>

\*"Planetary Satellites: An Update", Sky and Telescope, November 1983.

- 5. List the planets (that orbit the sun) in order, starting with the one closest to the sun and leaving Pluto off, since it's not "really" a planet anymore. You might need to use the internet as a resource.
- 6. According to Kepler's Laws, which planet takes longer to orbit the sun, Saturn or Neptune? Explain.
- 7. Calculate the Kepler constants by determining the  $R^3/T^2$  values for each of the planet systems listed in the chart at the bottom of the previous page.
- 8. An asteroid of diameter 100 km has a mean radius of orbit of  $4.8 \times 10^{11}$  m. What will be its orbital period around the sun?
- 9. A spy satellite is located one Earth radius above the surface of the Earth. What is its period of revolution?
- 10. Mars has two moons, Phobos and Deimos (Fear and Panic, the companions of Mars, the god of war). Deimos has a period of 30 h 18 min and a mean distance from the centre of Mars of  $2.3 \times 10^4$  km. If the period of Phobos is 7 h 39 min, what mean distance is it from the centre of Mars?
- 11. A Martian lander is to be placed in orbit around Mars at a mean altitude of 100 km. What will be the period of the Martian lander?
- 12. Communications satellites are placed in orbit so that they remain stationary relative to a specific area on the Earth's surface. They are given the name **synchronous satellites** because, to maintain such a position, their period as they orbit must be the same as the Earth's. What is the height of such a satellite measured from (a) the centre of the Earth, and (b) the surface of the Earth?

### ANSWERS

| 2,2,2,1,1                                               |                                                     |                                    |
|---------------------------------------------------------|-----------------------------------------------------|------------------------------------|
| 1) $\sim 3.35E18 \text{ m}^3/\text{s}^2$ for all except | 5) in class discussion                              | 10) 9.2 X 10° m                    |
|                                                         | -,                                                  | -)                                 |
| for the moon.                                           |                                                     |                                    |
|                                                         | 6) in along discussion                              |                                    |
| 7                                                       | 0) In class discussion                              |                                    |
| 2) $4.9 \times 10^{7} \text{ sec}$                      |                                                     |                                    |
|                                                         | 7)                                                  |                                    |
|                                                         | Earth: $1.01E13 \text{ m}^{3}/\text{s}^{2}$         |                                    |
|                                                         | Mars: $1.00 \text{ E}12 \text{ m}^{3}/\text{s}^{2}$ |                                    |
|                                                         |                                                     |                                    |
|                                                         | Jupiter: $3.21E15 \text{ m}^3/\text{s}^2$           |                                    |
|                                                         | Saturn: 9.64E14 $m^3/s^2$                           |                                    |
|                                                         | $1.45 \Gamma 1.4 m^3/r^2$                           |                                    |
|                                                         | Uranus: 1.45E14 m /s                                |                                    |
|                                                         | Neptune: $1.74E14 \text{ m}^3/\text{s}^2$           |                                    |
|                                                         | Pluto: $2.51 E 10 m^3/s^2$                          |                                    |
|                                                         | 11uto. 2.51L10 III /3                               |                                    |
| (3) 1.6 earth radii                                     |                                                     | 11) $6,352.6$ sec                  |
| ,                                                       | 9) 1.92 V $10^8$ cas                                | , ,                                |
|                                                         | $8) 1.82 \times 10$ sec                             |                                    |
|                                                         |                                                     | 12) 4.2 x $10^7$ m; 3.6 x $10^7$ m |
| 4) 1.01 V 10 <sup>13</sup> $m^{3}/c^{2}$                | 0) 1 42 V $10^4$ see                                | , , , , ,                          |
| 4) 1.01 A 10 III /S                                     | 9) 1.43 A 10 SEC                                    |                                    |