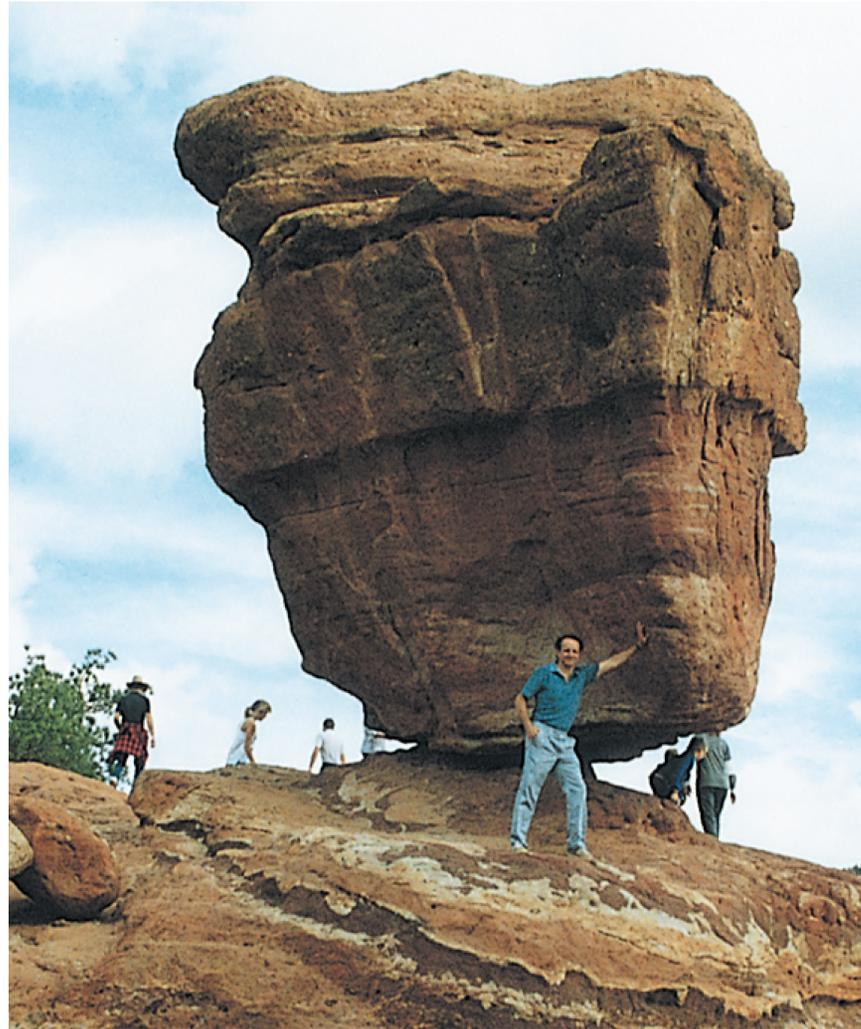


Topic 8: Rotational Equilibrium and Dynamics



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College Physics, 11e
Raymond A. Serway; Chris
Vuille

 BROOKS/COLE
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Reading Question 1-1

Which of the following screwdrivers would likely be the most effective to loosen a stubborn screw?

Assume the size of the driver head is the same in each case.

1. one with a handle that is 8 cm long and 2 cm wide
2. one with a handle that is 12 cm long and 2 cm wide
3. one with a handle that is 8 cm long and 4 cm wide
4. They would all loosen the screw just as effectively.

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2. one with a handle that is 12 cm long and 2 cm wide
- 3. one with a handle that is 8 cm long and 4 cm wide**
4. They would all loosen the screw just as effectively.

Reading Question 1-2

A ball rolls without slipping down incline A, starting from rest. At the same time, a box starts from rest and slides down incline B, which is identical to incline A except it is frictionless. Which has a larger translational speed at the bottom of the ramp?

1. They have the same translational speed at the bottom of the ramp.
2. the ball
3. the box
4. It is impossible to determine.

Reading Question 1-2

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2. the ball
- 3. the box**
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Reading Question 1-3

Complete the following sentence: "The moment of inertia of an object

1. depends on the rotational speed of the object."
2. will be greater for a larger-massed object than for a smaller-massed object."
3. is not a function of the object's mass."
4. is a function of the axis about which the object is spun."

Reading Question 1-3

Complete the following sentence: "The moment of inertia of an object

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3. is not a function of the object's mass."
4. **is a function of the axis about which the object is spun."**

Reading Question 1-4

A skater rotates quickly on the ice. He pushes his arms out. What happens to his angular speed?

1. It increases.
2. It decreases.
3. It stays the same.
4. It is impossible to determine.

Reading Question 1-4

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1. It increases.
- 2. It decreases.**
3. It stays the same.
4. It is impossible to determine.

Reading Question 1-5

Someone hits a tether ball to make it spin around a pole. The cord that attaches the tether ball to the central pole then winds around the pole. What happens to the angular speed of the tether ball during this process?

1. It decreases.
2. It increases.
3. It remains constant.
4. It cannot be determined.

Reading Question 1-5

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1. It decreases.
- 2. It increases.**
3. It remains constant.
4. It cannot be determined.

Reading Question 1-6

The center of gravity is a useful point at which to balance an object. For instance, a ruler can be balanced by placing its center on your finger. Is it possible for the center of gravity of an object to be located at a point where no material exists to balance the object?

1. no, never
2. only for very unsymmetrical objects
3. It is possible in theory, but no objects have been found for which this occurs.
4. Yes, it is possible.

Reading Question 1-6

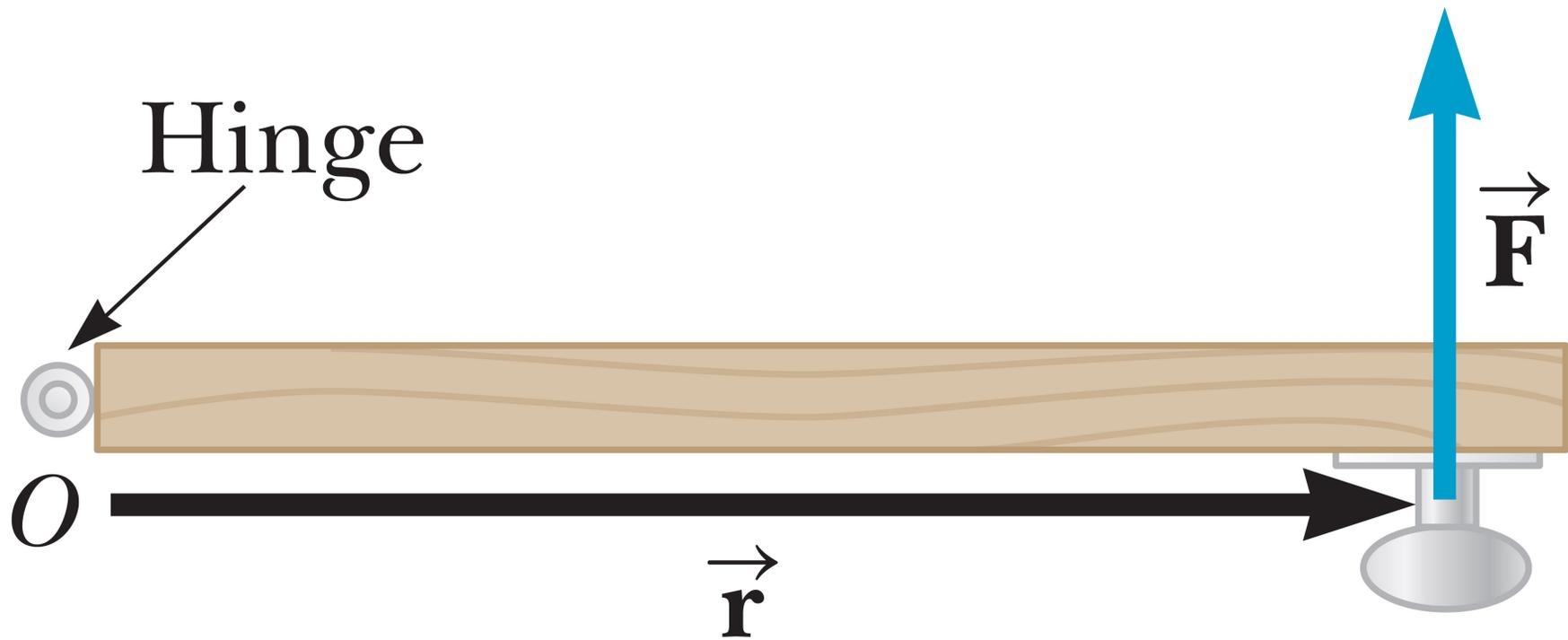
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3. It is possible in theory, but no objects have been found for which this occurs.
4. **Yes, it is possible.**

Topic 8: Rotational Equilibrium and Dynamics

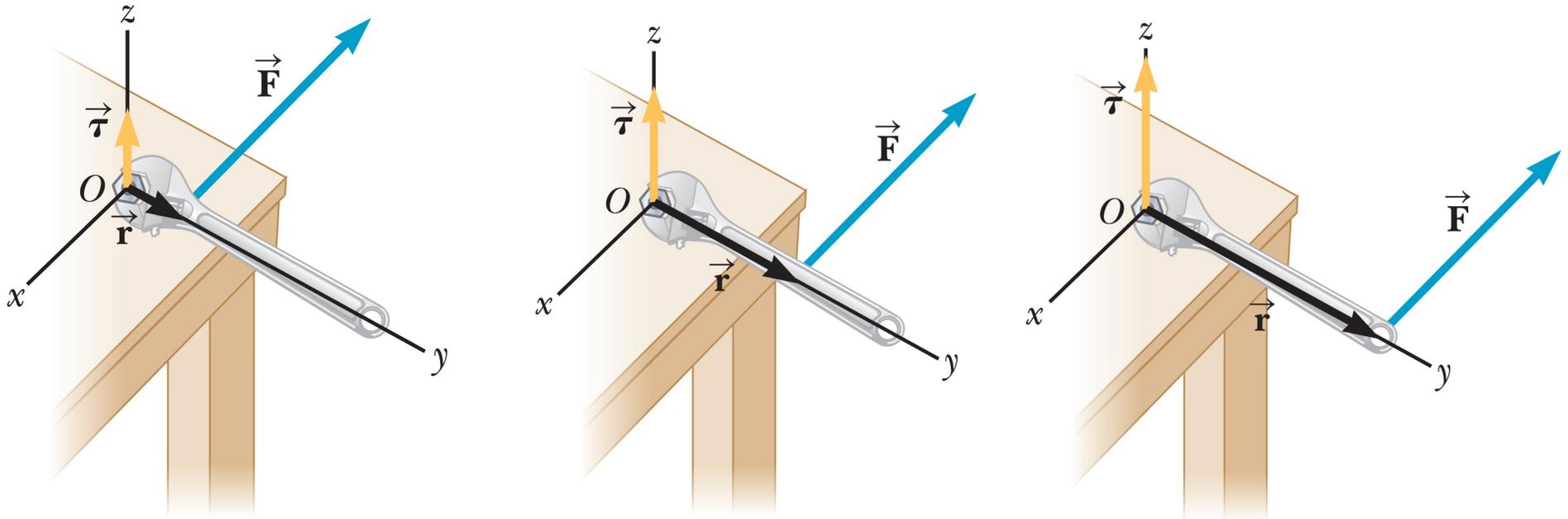
TOPIC DISCUSSION

Torque



$$\tau = rF \quad \text{SI unit: N} \cdot \text{m}$$

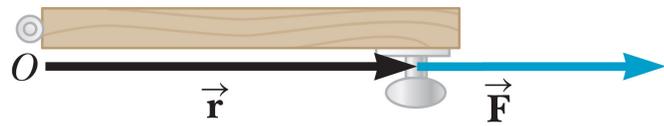
Torque



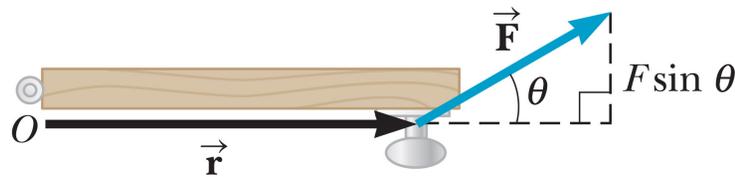
- Counterclockwise \rightarrow positive direction
- Clockwise \rightarrow negative direction

The rate of rotation of an object doesn't change, unless the object is acted on by a net torque.

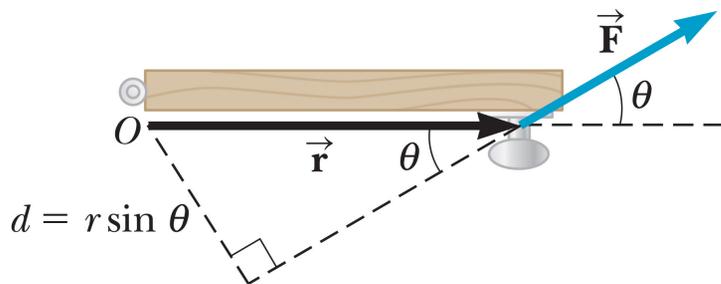
Torque



a



b



c

$$F_{\text{perpendicular}} = F \sin \theta$$

for $\theta = 0^\circ$:

$$\sin 0^\circ = 0, F \sin \theta = 0$$

for $\theta = 180^\circ$:

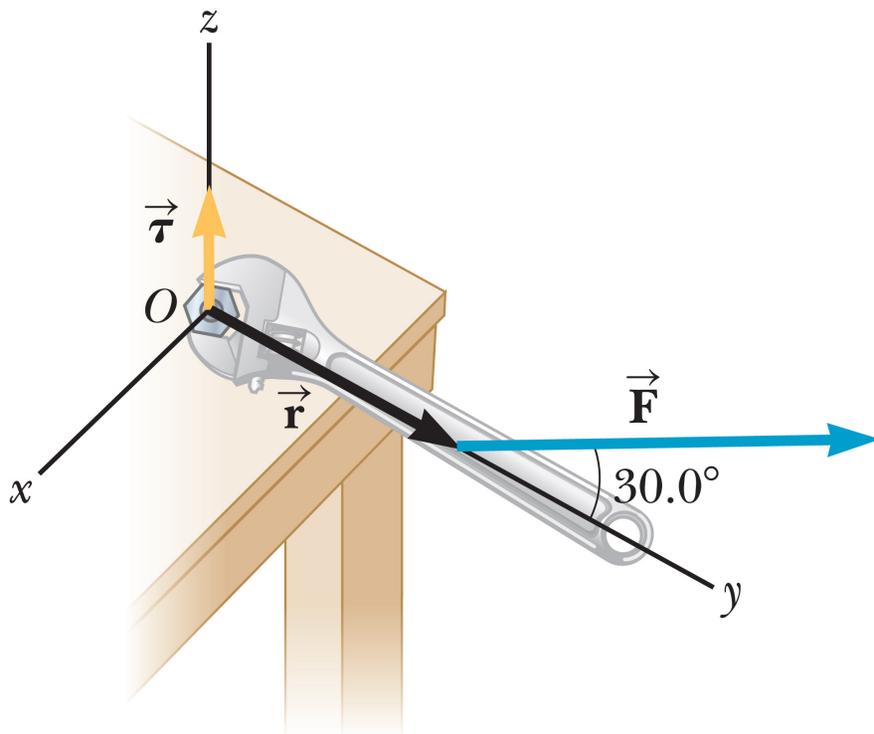
$$\sin 180^\circ = 0, F \sin \theta = 0$$

for $\theta = 90^\circ$ or 270° :

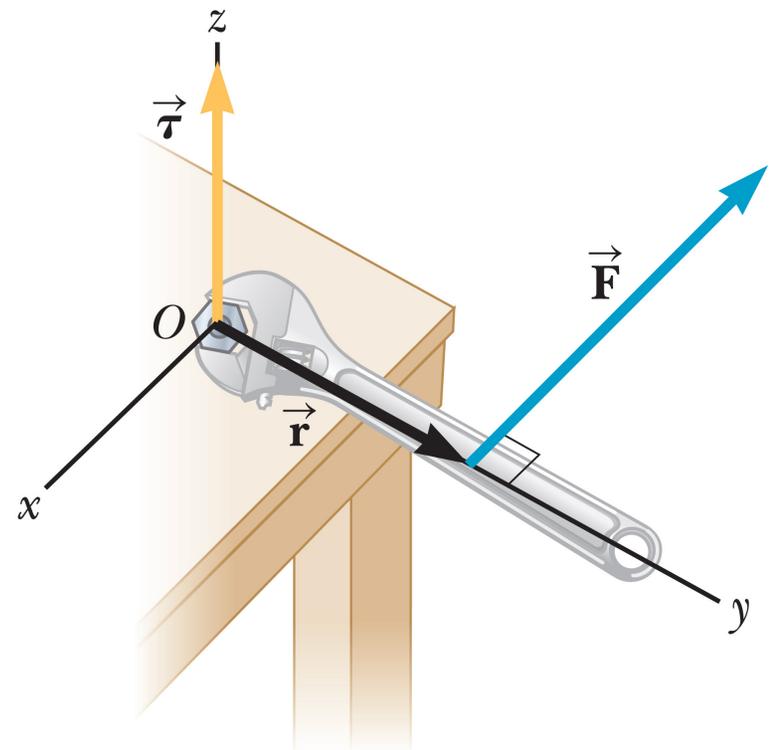
$$\sin \theta = 1, F \sin \theta = F$$

$$\tau = rF \sin \theta$$

Torque



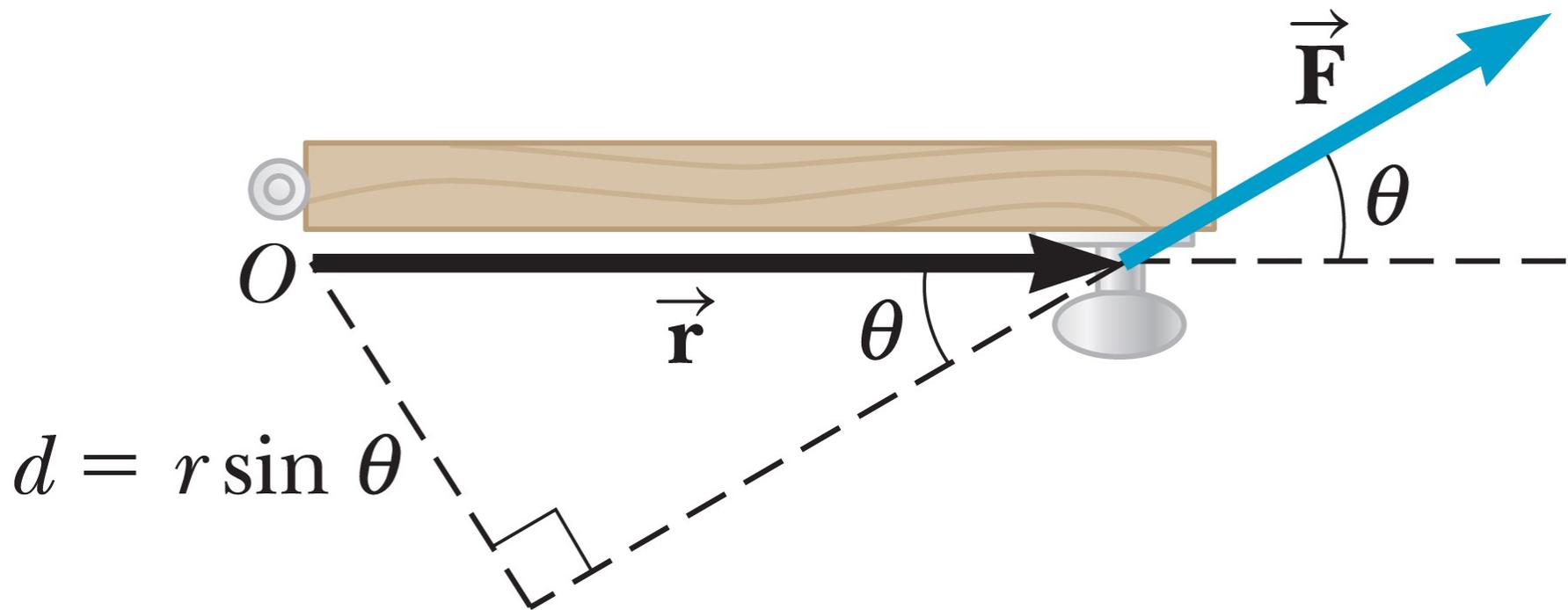
a



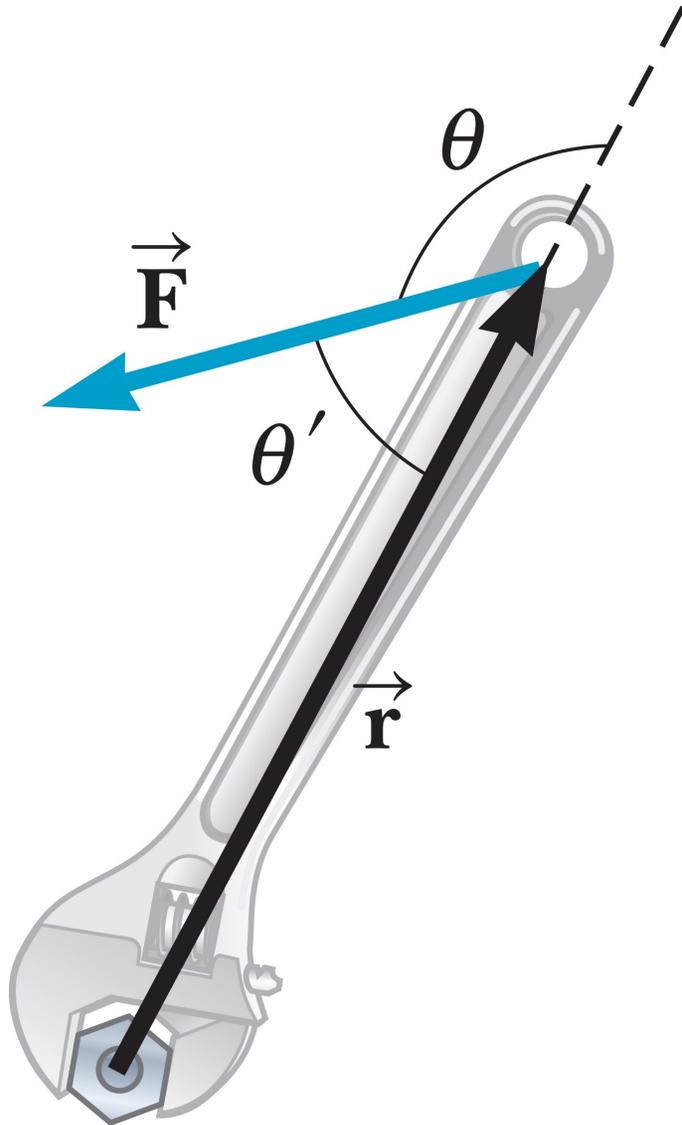
b

Torque

$$d = r \sin \theta$$



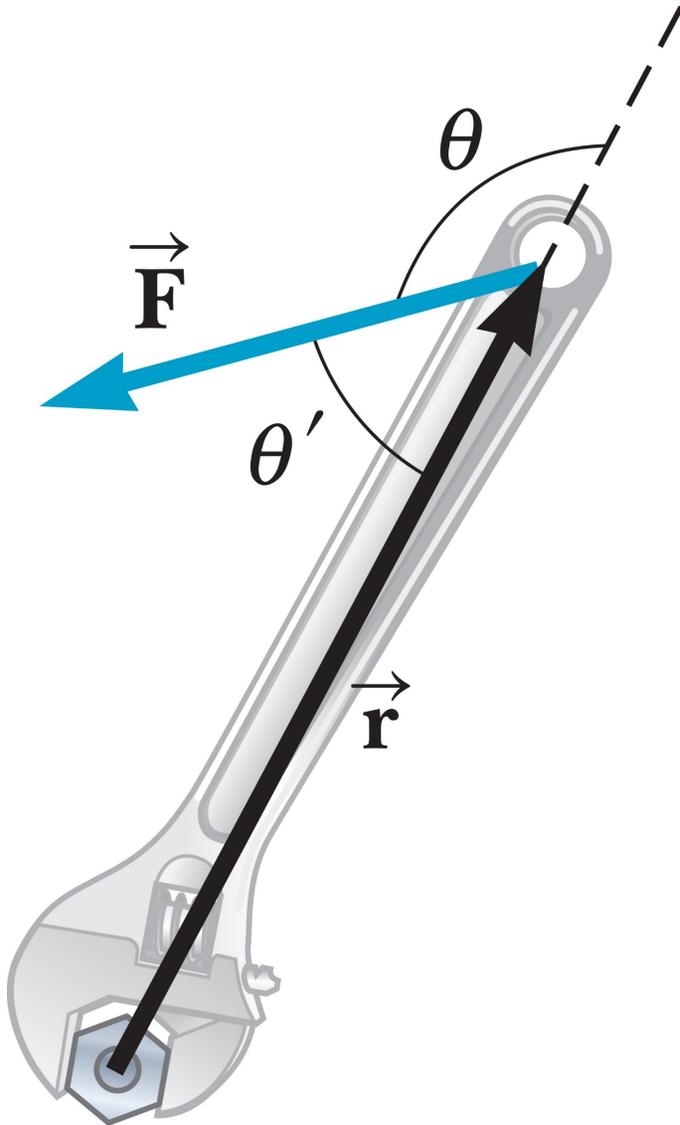
Torque



Right-Hand Rule:

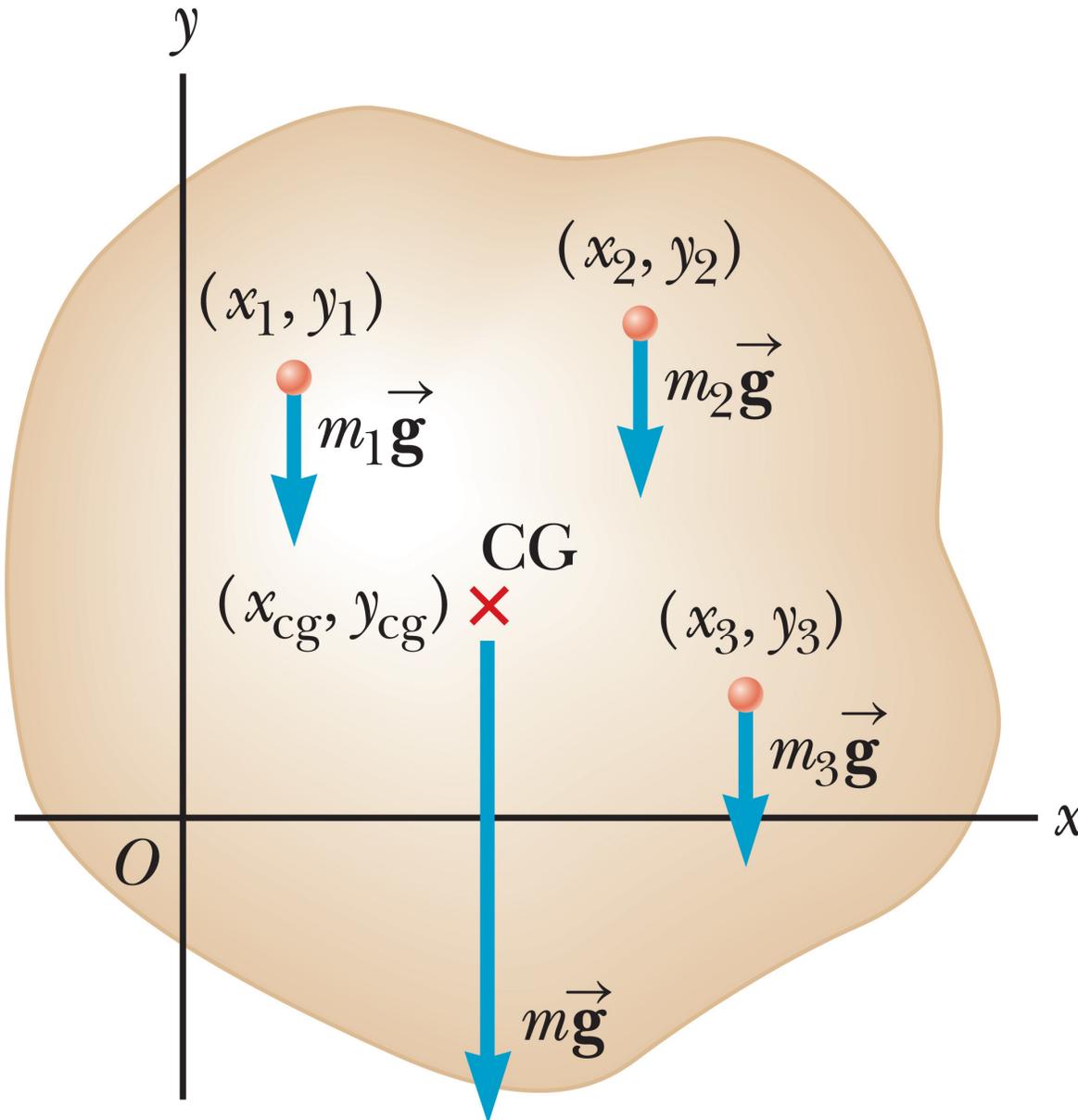
1. Point the fingers of your right hand in the direction of \vec{r} .
2. Curl your fingers toward the direction of vector \vec{F} .
3. Your thumb then points approximately in the direction of the torque.

Torque



$$\begin{aligned}\sin \theta &= \sin (180^\circ - \theta) \\ &= \sin (180^\circ) \cos \theta - \sin \theta \cos (180^\circ) \\ &= 0 - \sin \theta \cdot (-1) \\ &= \sin \theta\end{aligned}$$

Center of Mass



$$\tau_1 = m_1 g x_1$$

$$w = F_g = Mg$$

Center of Mass

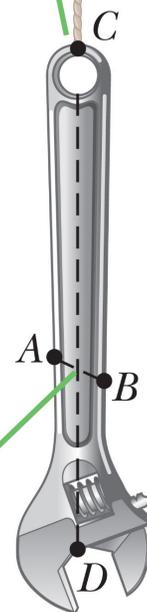
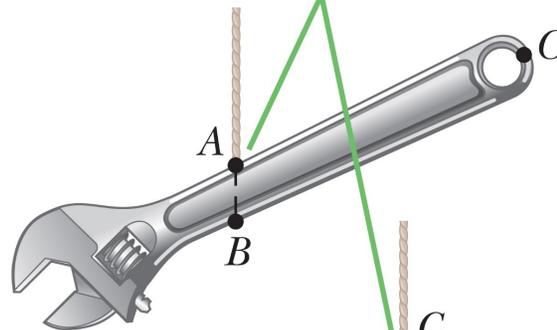
$$(m_1g + m_2g + m_3g + \dots)x_{cg} = m_1gx_1 + m_2gx_2 + m_3gx_3 + \dots$$

$$x_{cg} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots}{m_1 + m_2 + m_3 + \dots} = \frac{\Sigma m_i x_i}{\Sigma m_i}$$

$$y_{cg} = \frac{\Sigma m_i y_i}{\Sigma m_i} \quad z_{cg} = \frac{\Sigma m_i z_i}{\Sigma m_i}$$

Center of Mass

The wrench is hung freely first from point A and then from point C .



The intersection of the two lines AB and CD locates the center of mass.

Center of Mass Motion

$$M_{\text{tot}} \vec{a}_{\text{cm}} = \Sigma \vec{F}_i = \Sigma \vec{F}_{\text{ext}} + \Sigma \vec{F}_{\text{int}}$$

$$M_{\text{tot}} \vec{a}_{\text{cm}} = \Sigma \vec{F}_{\text{ext}}$$

$$x_{\text{cm}} = \frac{1}{2} a_{\text{cm}} t^2 + v_{0,\text{cm}} t + x_{0,\text{cm}}$$

$$v_{\text{cm}} = a_{\text{cm}} t + v_{0,\text{cm}}$$

$$v_{\text{cm}}^2 - v_{0,\text{cm}}^2 = 2a_{\text{cm}} \Delta x_{\text{cm}}$$

Torque and the Two Conditions for Equilibrium

1. The net external force must be zero: $\Sigma \vec{F} = 0$
2. The net external torque must be zero: $\Sigma \vec{\tau} = 0$

Problem-Solving Strategy: Objects in Equilibrium

1. Diagram the system
2. Draw a force diagram
3. Apply the second condition of equilibrium

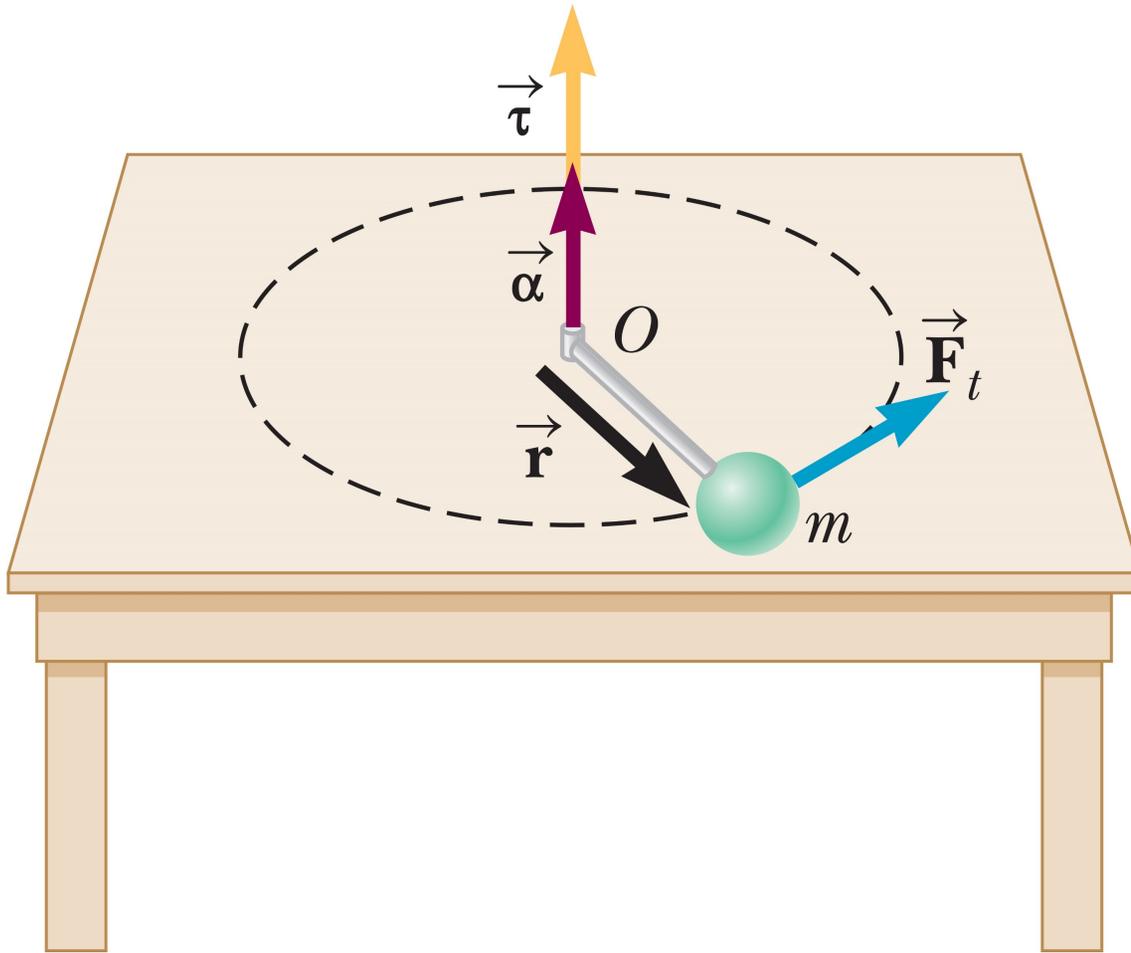
$$\Sigma \tau_i = 0$$

4. Apply the first condition of equilibrium

$$\Sigma F_x = 0 \quad \Sigma F_y = 0$$

5. Solve the system of equations

The Rotational Second Law of Motion



$$F_t = ma_t$$

$$F_t r = mra_t$$

$$a_t = r\alpha \rightarrow F_t = ma_t$$

$$\rightarrow F_t r = mr^2 \alpha$$

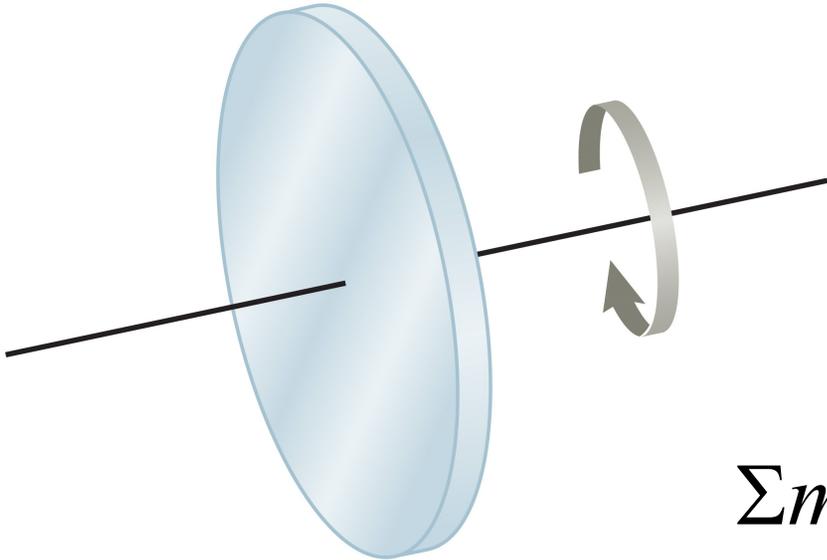
$$\tau = mr^2 \alpha$$

Think – Pair – Share

Using a screwdriver, you try to remove a screw from a piece of furniture, but can't get it to turn. To increase the chances of success, you should use a screwdriver that

1. is longer.
2. is shorter.
3. has a narrower handle.
4. has a wider handle.

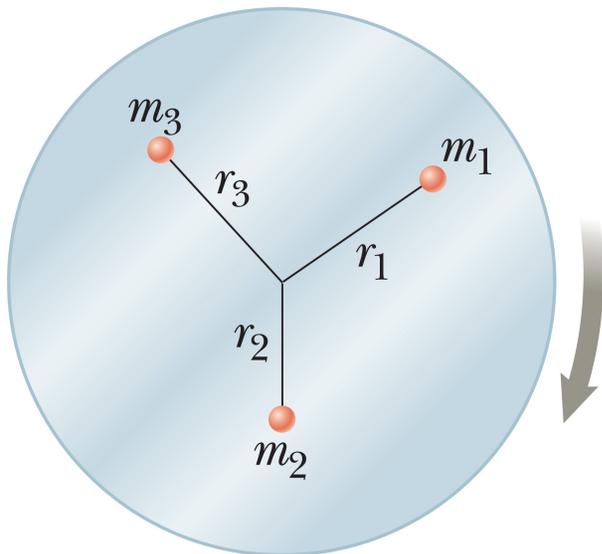
Torque on a Rotating Object



$$\tau = mr^2\alpha$$

$$\Sigma\tau = (\Sigma mr^2)\alpha$$

$$\Sigma mr^2 = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \dots$$



$$I \equiv \Sigma mr^2 \quad \text{SI units: kg} \cdot \text{m}^2$$

$$\Sigma\tau = I\alpha$$

Think – Pair – Share

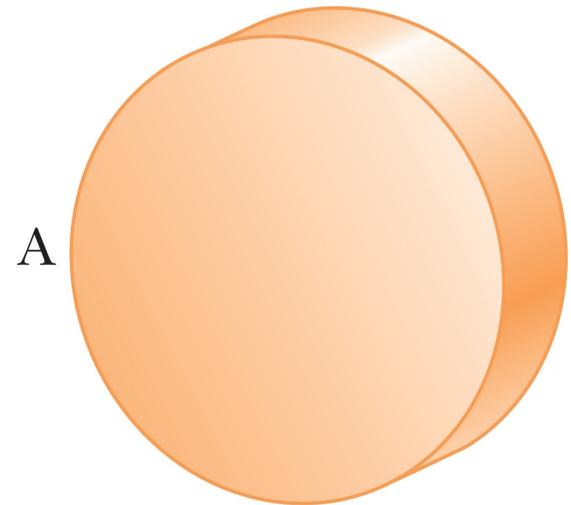
A constant net torque is applied to an object. Which one of the following will *not* be constant?

1. angular acceleration
2. angular velocity
3. moment of inertia
4. center of gravity

Think – Pair – Share

The two rigid objects shown in the figures have the same mass, radius, and angular speed. If the same braking torque is applied to each, which takes longer to stop?

1. A
2. B
3. more information is needed

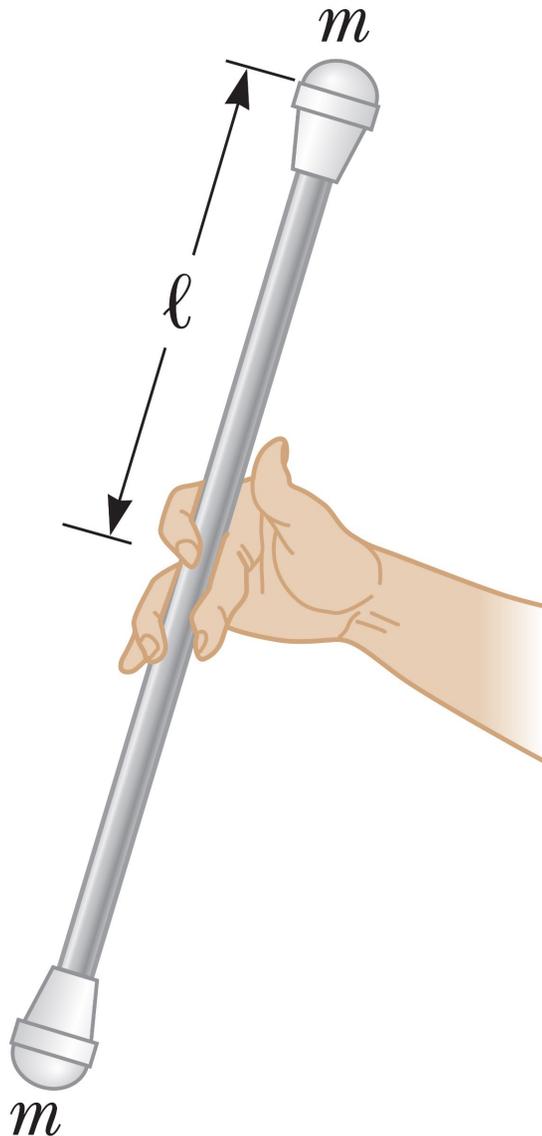


Torque on a Rotating Object



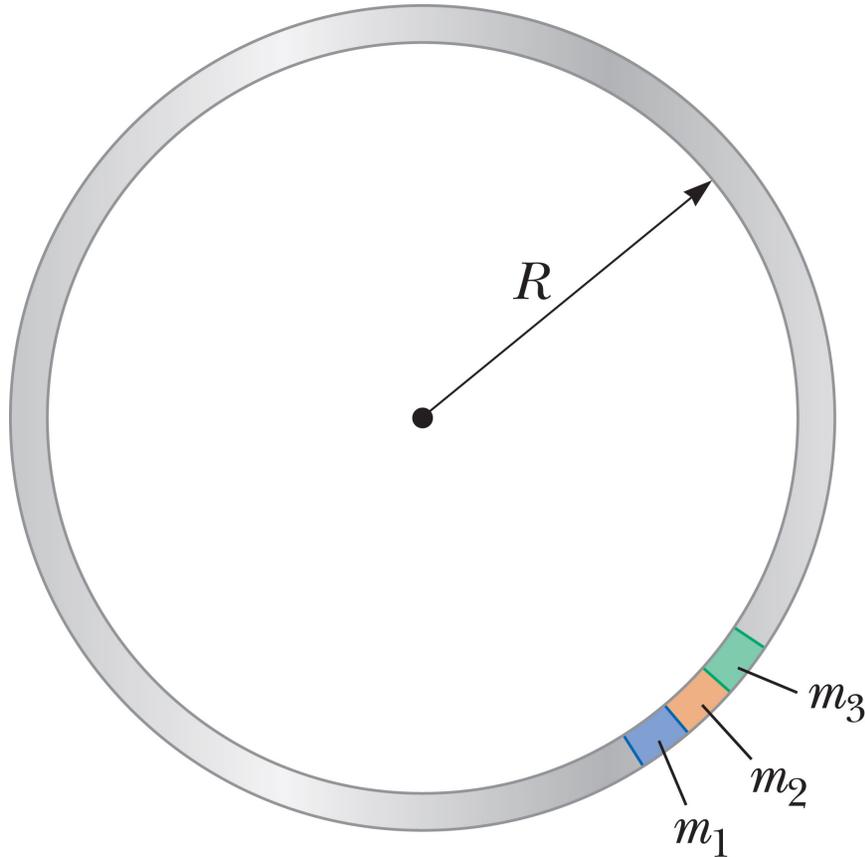
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More on the Moment of Inertia



$$\begin{aligned} I &= \Sigma mr^2 \\ &= ml^2 + ml^2 \\ &= 2ml^2 \end{aligned}$$

Calculations of Moments of Inertia for Extended Objects



$$I = \Sigma mr^2$$

$$M = m_1 + m_2 + m_3 + \dots + m_n$$

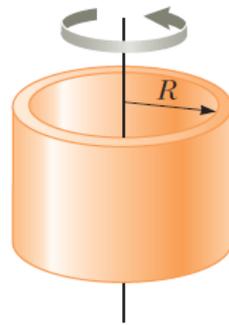
$$I = \Sigma mr^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2$$

©Cengage
$$I = (m_1 + m_2 + m_3 + \dots + m_n) R^2 = MR^2$$

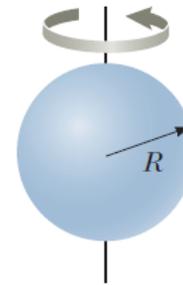
Calculations of Moments of Inertia for Extended Objects

Table 8.1 Moments of Inertia for Various Rigid Objects of Uniform Composition

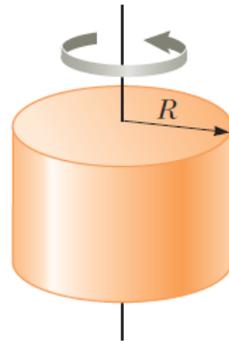
Hoop or thin cylindrical shell
 $I = MR^2$



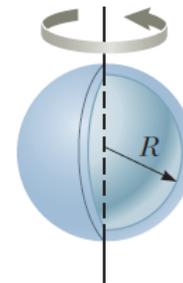
Solid sphere
 $I = \frac{2}{5} MR^2$



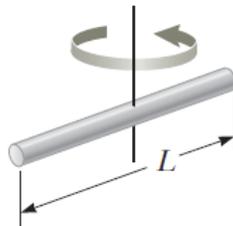
Solid cylinder or disk
 $I = \frac{1}{2} MR^2$



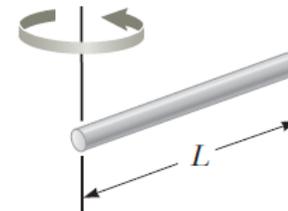
Thin spherical shell
 $I = \frac{2}{3} MR^2$



Long, thin rod with rotation axis through center
 $I = \frac{1}{12} ML^2$



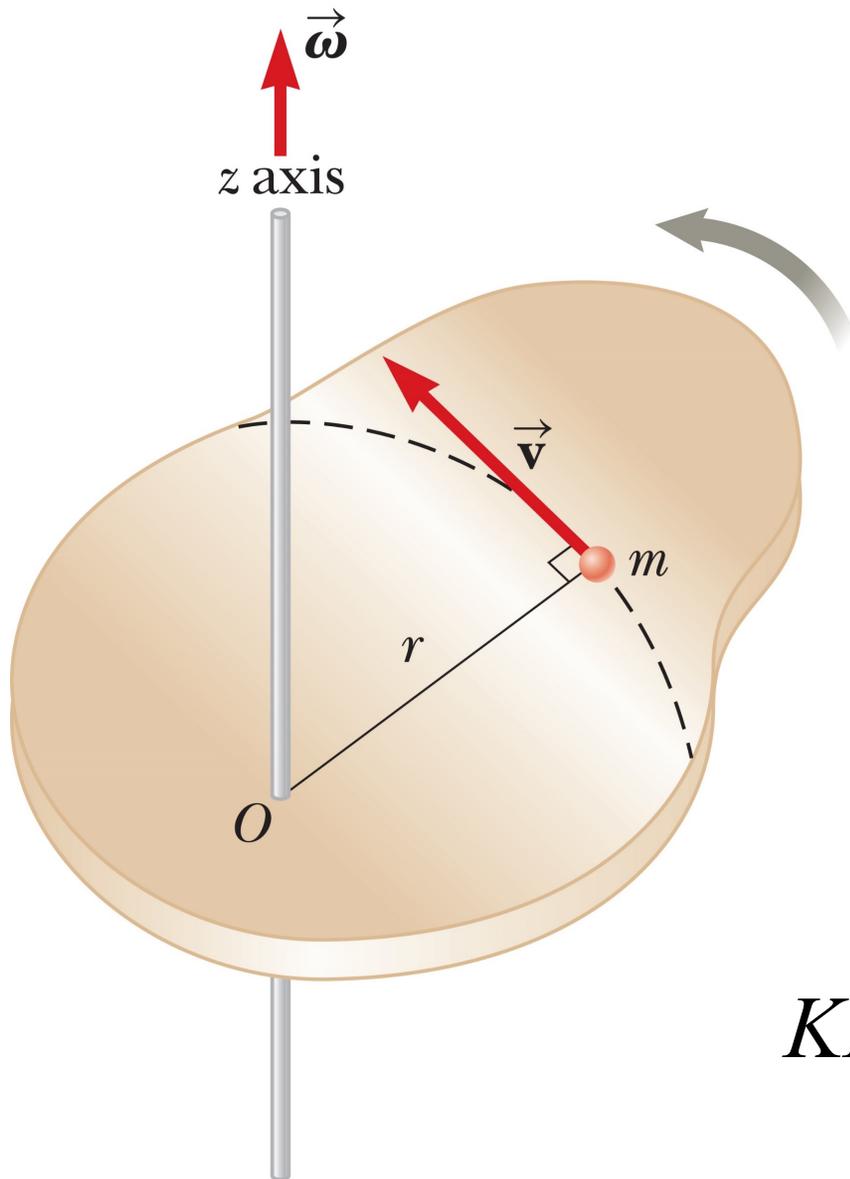
Long, thin rod with rotation axis through end
 $I = \frac{1}{3} ML^2$



Extending the System Approach

$$\left(\Sigma m_i + \Sigma \frac{I_i}{r_i^2} \right) a = \Sigma F_{\text{ext}}$$

Rotational Kinetic Energy



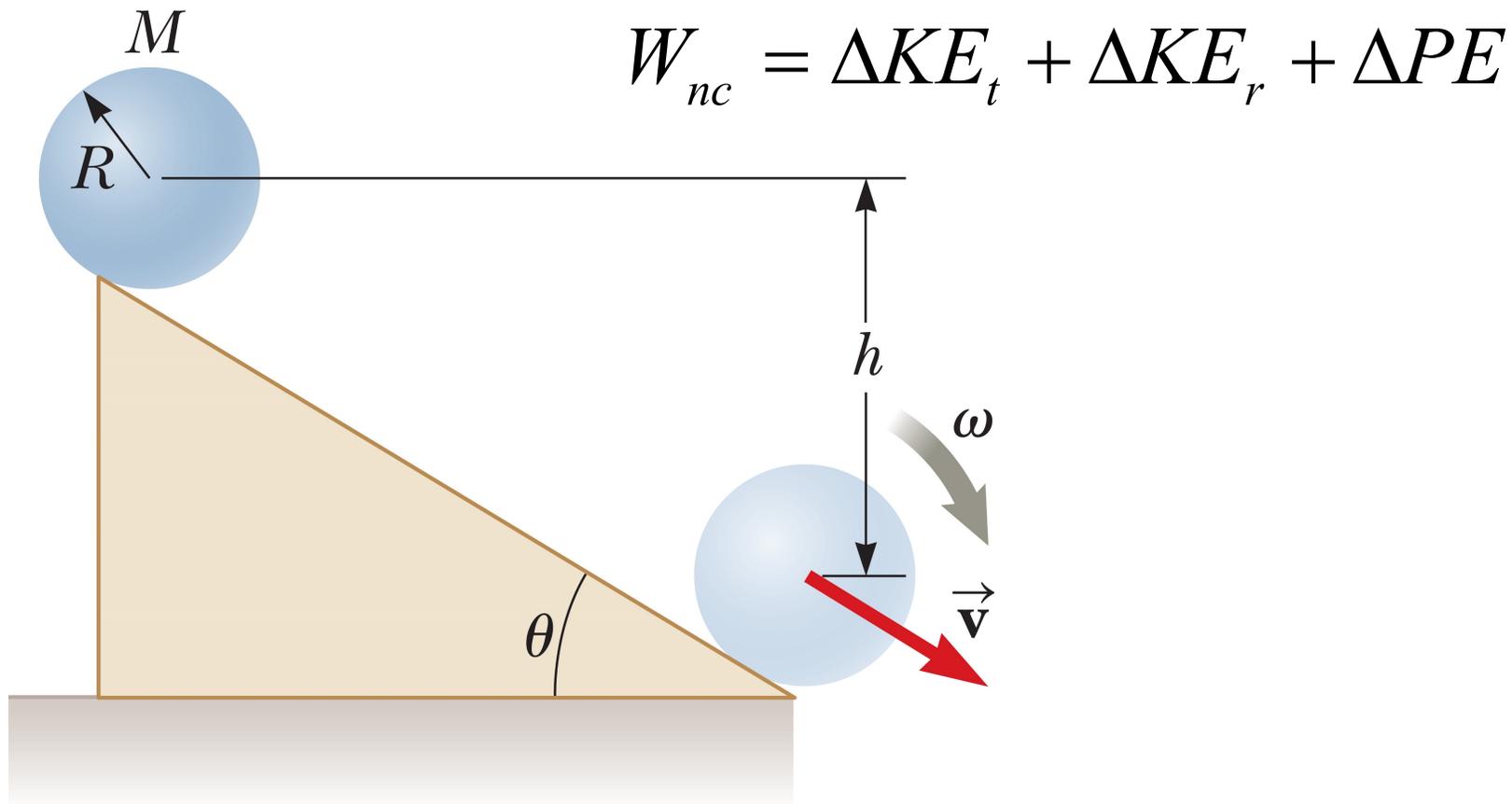
$$v = r\omega$$

$$\begin{aligned} KE_r &= \sum \left(\frac{1}{2} I \omega^2 \right) \\ &= \sum \left(\frac{1}{2} m r^2 \omega^2 \right) \\ &= \frac{1}{2} \sum (m r^2) \omega^2 \end{aligned}$$

$$KE_r = \frac{1}{2} I \omega^2 \quad \text{with } I = \sum m r^2$$

Rotational Kinetic Energy

$$(KE_t + KE_r + PE)_i = (KE_t + KE_r + PE)_f$$



Problem-Solving Strategy: Energy Methods and Rotation

1. Choose two points of interest
2. Identify forces
3. Write the appropriate equation:

$$(KE_t + KE_r + PE)_i = (KE_t + KE_r + PE)_f$$

$$W_{nc} = \Delta KE_t + \Delta KE_r + \Delta PE$$

4. Substitute general expressions
5. Use

$$v = r\omega$$

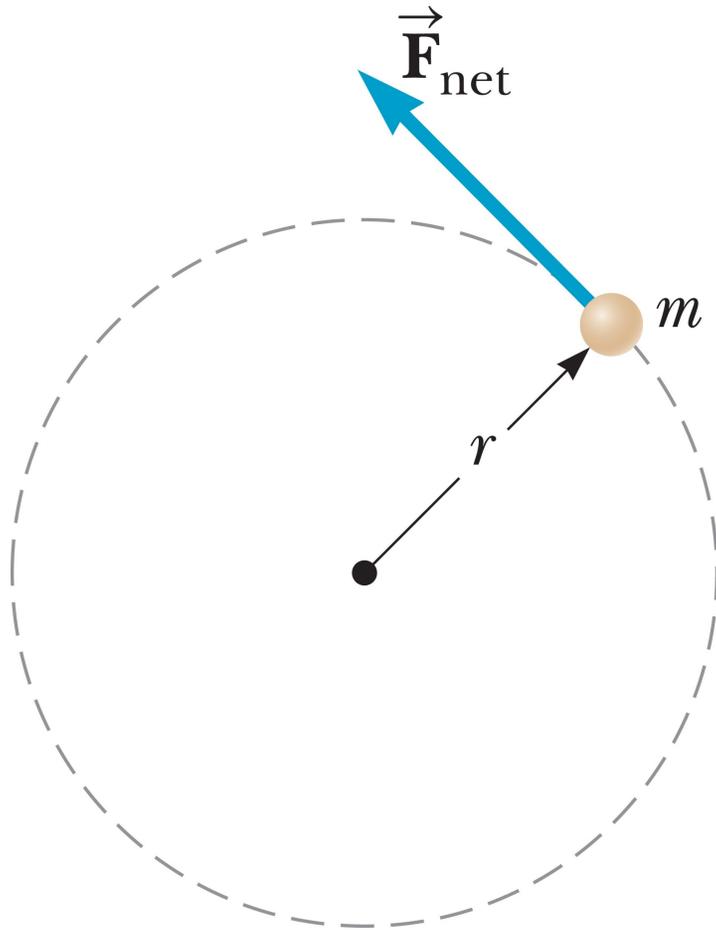
6. Solve for the unknown

Think – Pair – Share

Two spheres, one hollow and one solid, are rotating with the same angular speed around an axis through their centers. Both spheres have the same mass and radius. Which sphere, if either, has the higher rotational kinetic energy?

1. the hollow sphere
2. the solid sphere
3. They have the same kinetic energy.

Angular Momentum



$$\Sigma \tau = I \alpha = I \frac{\Delta \omega}{\Delta t}$$

$$= I \left(\frac{\omega - \omega_0}{\Delta t} \right)$$

$$= \frac{I \omega - I \omega_0}{\Delta t}$$

$$L \equiv I \omega$$

$$\Sigma \tau = \frac{\text{change in angular momentum}}{\text{time interval}} = \frac{\Delta L}{\Delta t}$$

Angular Momentum

$$\frac{\Delta L}{\Delta t} = 0$$

$$\Sigma \tau = 0 \quad L_i = L_f$$

$$I_i \omega_i = I_f \omega_f \quad \text{if } \Sigma \tau = 0$$

The mechanical energy, linear momentum, and angular momentum of an isolated system all remain constant.

Angular Momentum

By pulling in his arms and legs, he reduces his moment of inertia and increases his angular speed (rate of spin).



Clive Rose/Getty Images

Upon landing, extending his arms and legs increases his moment of inertia and helps slow his spin.



Al Bello/Getty Images

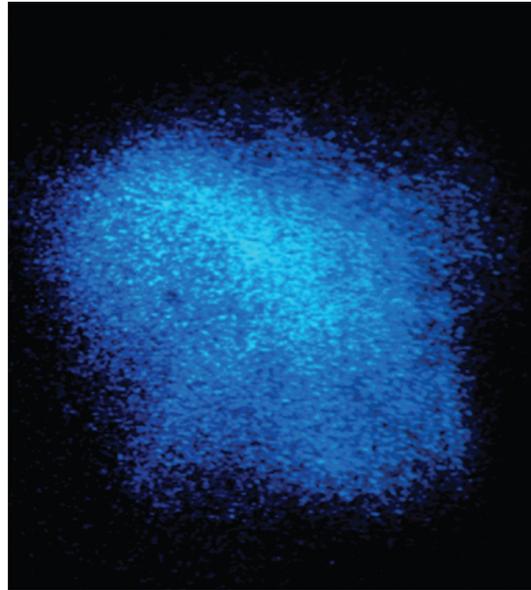
Angular Momentum

Max Planck Institute for Astronomy,
Heidelberg, Germany



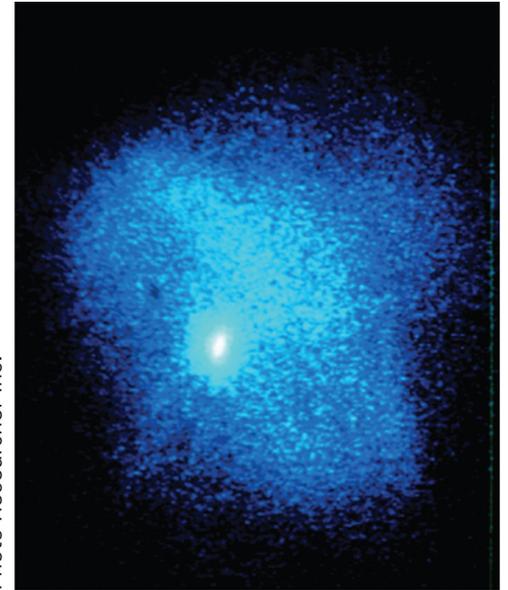
a

Smithsonian Institution/Science Source/
Photo Researcher Inc.



b

Smithsonian Institution/Science Source/
Photo Researcher Inc.



c

Think – Pair – Share

A horizontal disk with moment of inertia I_1 rotates with angular speed ω_1 about a vertical frictionless axle. A second horizontal disk, with moment of inertia I_2 drops onto the first, initially not rotating but sharing the same axis as the first disk. Because their surfaces are rough, the two disks eventually reach the same angular speed ω . The ratio ω/ω_1 is equal to

1. I_1 / I_2
2. I_2 / I_1
3. $I_1 / (I_1 + I_2)$
4. $I_2 / (I_1 + I_2)$

Think – Pair – Share

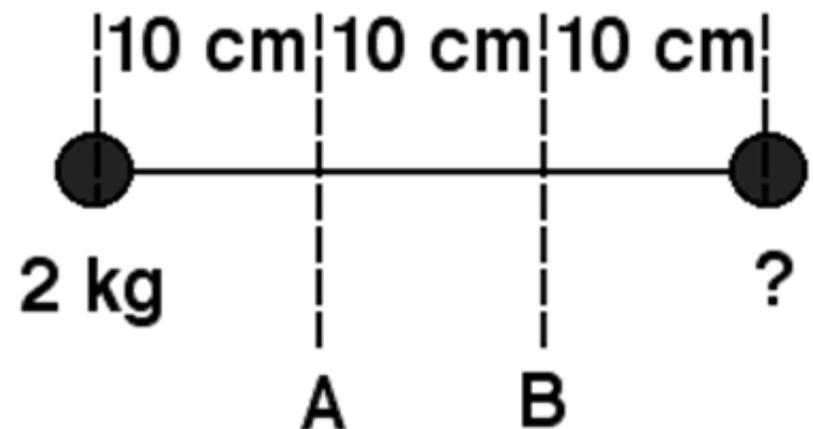
If global warming continues, it's likely that some ice from the polar ice caps of the Earth will melt and the water will be distributed closer to the equator. If this occurs, would the length of the day (one rotation)

1. increase.
2. decrease.
3. remain the same.

Assessing to Learn

The rotational inertia of the dumbbell (see figure) about axis A is twice the rotational inertia about axis B. The unknown mass is:

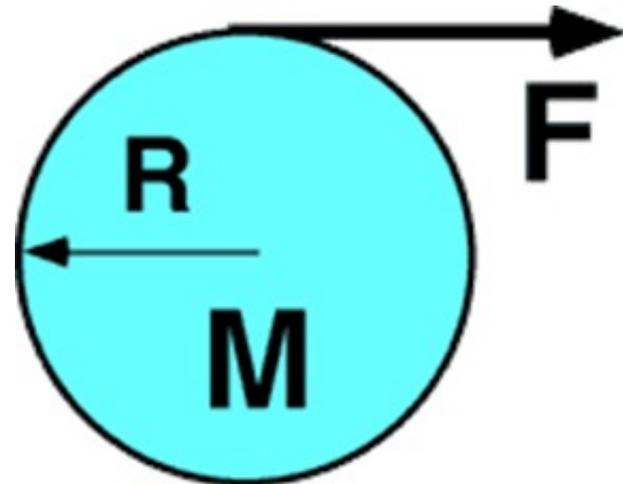
1. $\frac{4}{7}$ kg
2. 2 kg
3. 4 kg
4. 5 kg
5. 7 kg
6. 8 kg
7. 10 kg
8. None of the above
9. Cannot be determined
10. The rotational inertia cannot be different about different axes.



Assessing to Learn

A disk, with radius 0.25 m and mass 4 kg, lies flat on a smooth horizontal tabletop. A string wound about the disk is pulled with a force of 8 N. What is the acceleration of the disk?

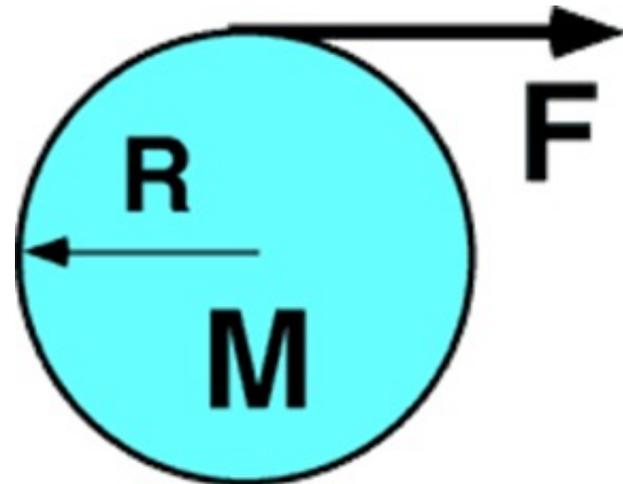
1. 0
2. 0.5 m/s^2
3. 1 m/s^2
4. 2 m/s^2
5. 4 m/s^2
6. None of the above.
7. Cannot be determined.



Assessing to Learn

A disk, with radius 0.25 m and mass 4 kg, lies flat on a smooth *horizontal* tabletop. A string wound about the disk is pulled with a force of 8 N. What is the angular acceleration of the disk?

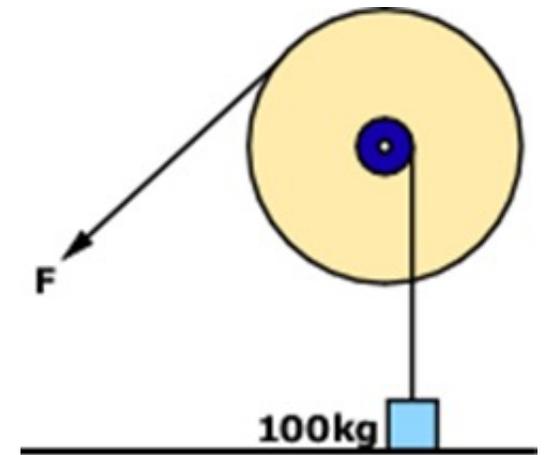
1. 0
2. 0.5 m/s^2
3. 1 m/s^2
4. 2 m/s^2
5. 4 m/s^2
6. None of the above.
7. Cannot be determined.



Assessing to Learn

A 100 kg crate is attached to a rope wrapped around the inner disk as shown. A person pulls on another rope wrapped around the outer disk with force F to lift the crate. What force F is needed to lift the crate 2 m?

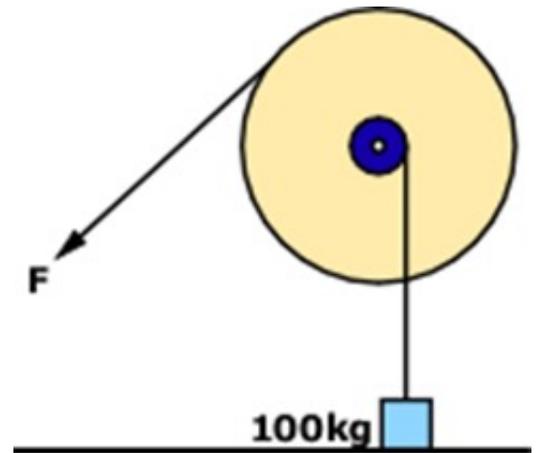
1. about 20 N
2. about 50 N
3. about 100 N
4. about 200 N
5. about 500 N
6. about 1,000 N
7. about 2,000 N
8. about 5,000 N
9. Impossible to determine without knowing the radii
10. Impossible to determine for some other reason(s)



Assessing to Learn

A 100-kg crate is attached to a rope wrapped around the inner disk as shown. A person pulls on another rope wrapped around the outer disk with force F to lift the crate. What force F is needed to lift the crate 2 m?

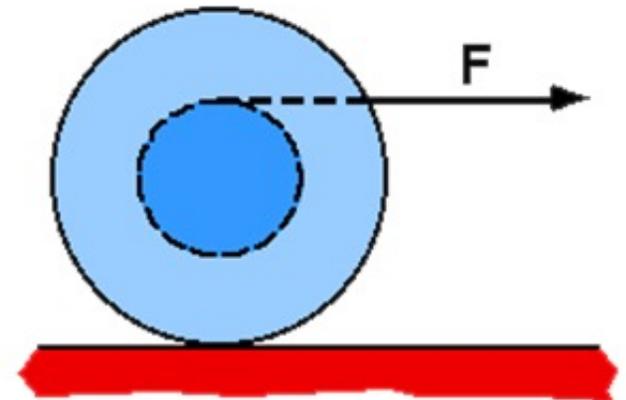
1. about 400 J
2. slightly less than 2,000 J
3. Exactly 2,000 J
4. slightly more than 2,000 J
5. much more than 2,000 J
6. Impossible to determine without knowing F
7. Impossible to determine without knowing the radii
8. Impossible to determine without knowing the mass of the pulley
9. Impossible to determine for two or more of the reasons given in 6, 7, and 8 above
10. Impossible to determine for some other reason(s)



Assessing to Learn

A spool has string wrapped around its center axle and is sitting on a horizontal surface. If the string is pulled in the horizontal direction when tangent to the top of the axle, the spool will...

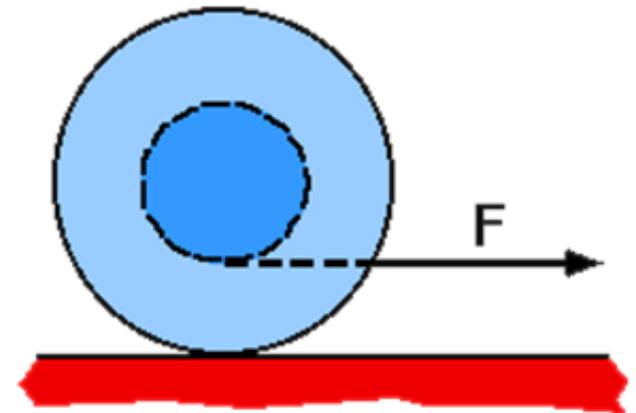
1. ... roll to the right.
2. ... not roll, only slide to the right.
3. ... spin and slip, without moving left or right.
4. ... roll to the left.
5. None of the above
6. The motion cannot be determined.



Assessing to Learn

A spool has string wrapped around its center axle and is sitting on a horizontal surface. If the string is pulled in the horizontal direction when tangent to the bottom of the axle, the spool will...

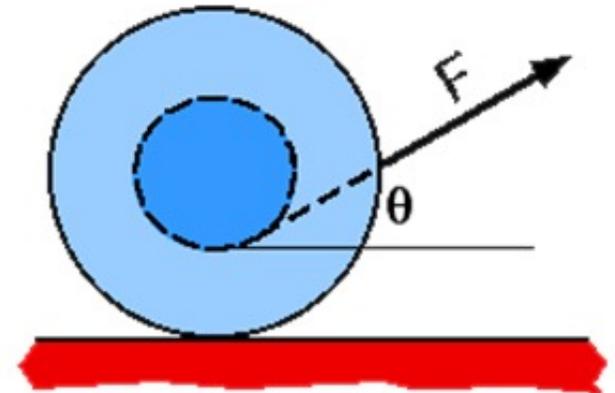
1. ... roll to the right.
2. ... not roll, only slide to the right.
3. ... spin and slip, without moving left or right.
4. ... roll to the left.
5. None of the above
6. The motion cannot be determined.



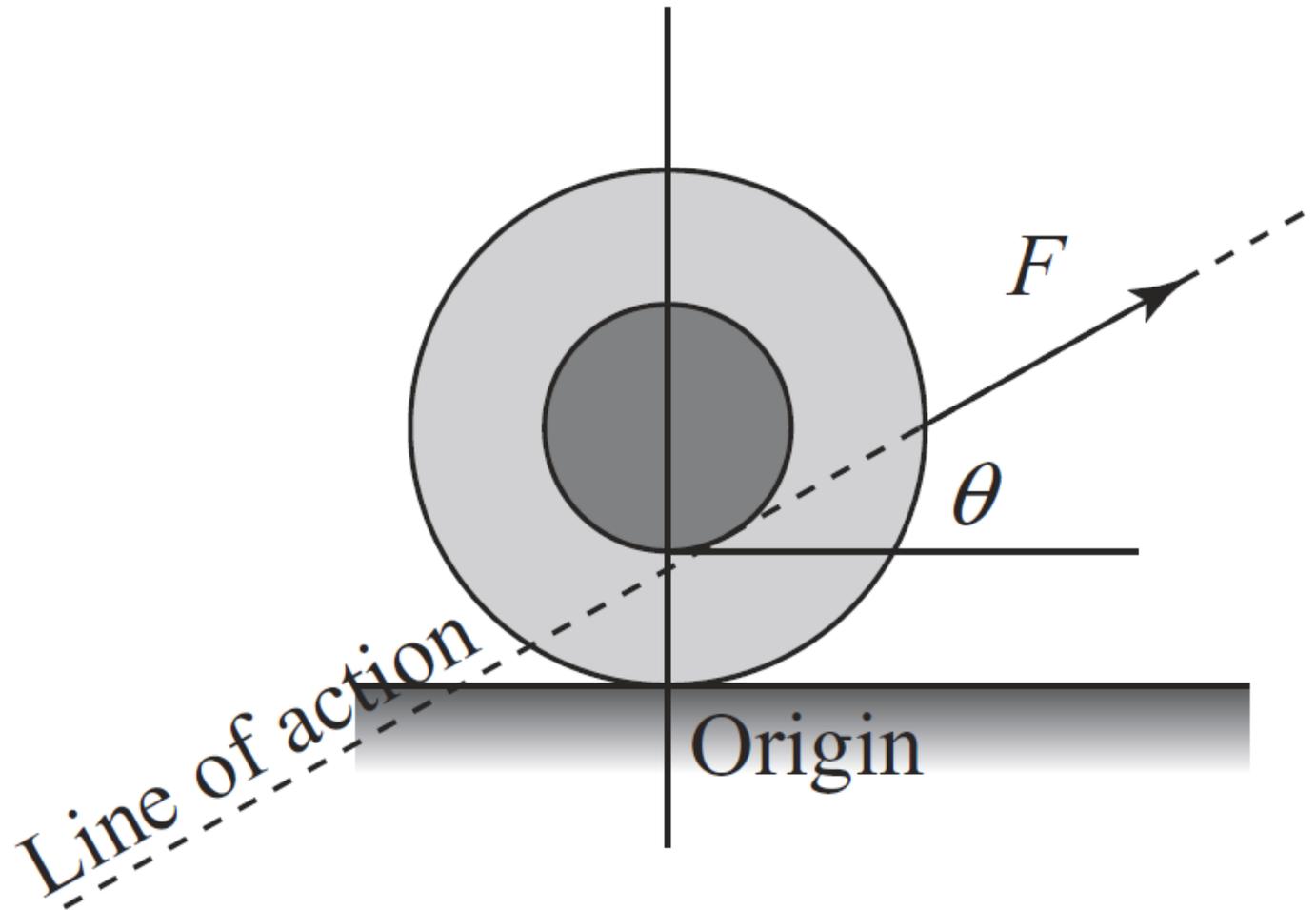
Assessing to Learn

A spool has string wrapped around its center axle and is sitting on a horizontal surface. If the string is pulled at an angle to the horizontal when drawn from the bottom of the axle, the spool will...

1. ... roll to the right.
2. ... not roll, only slide to the right.
3. ... spin and slip, without moving left or right.
4. ... roll to the left.
5. None of the above
6. The motion cannot be determined.



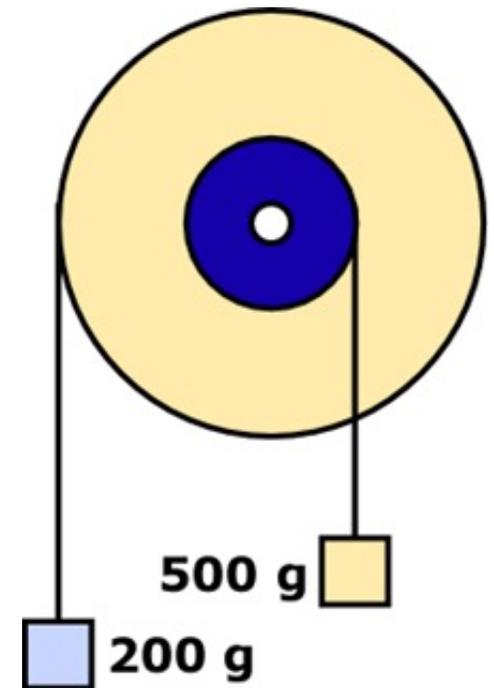
Assessing to Learn



Assessing to Learn

Two blocks hang from strings wound around different parts of a double pulley as shown. Assuming the system is **not** in equilibrium, what happens to the system's *potential energy* when it is released from rest?

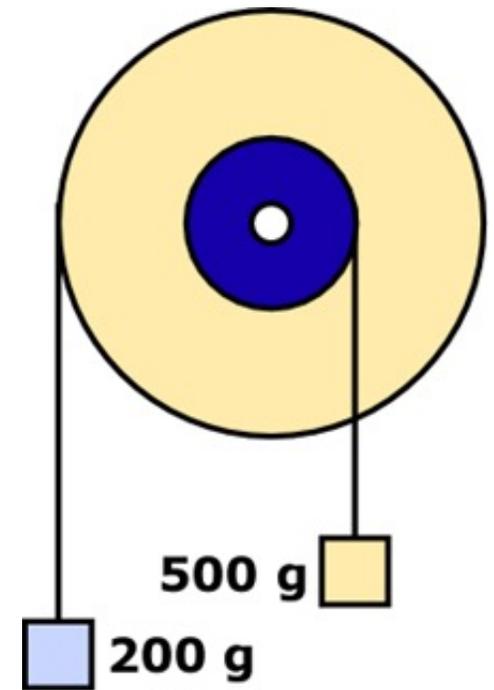
1. It remains the same.
2. It decreases.
3. It increases.
4. Impossible to determine without knowing the radii of the two pulleys.
5. Impossible to determine without knowing the ratio of the radii of the two pulleys.
6. Impossible to determine for some other reason.



Assessing to Learn

Two blocks hang from strings wound around different parts of a 2 kg double pulley as shown. The pivot exerts a normal force F_N supporting the double pulley. Assuming the system is **not** in equilibrium, which statement about F_N is true after the system is released from rest? (Use $g = 10 \text{ N/kg}$.)

1. $F_N = 20 \text{ N}$
2. $20 \text{ N} < F_N < 27 \text{ N}$
3. $F_N = 27 \text{ N}$
4. $F_N > 27 \text{ N}$
5. It is impossible to predict what the normal force on the double pulley will be.



Assessing to Learn

A uniform rod of length L , mass M , is suspended by two thin strings. Which of the following statements is true regarding the tensions in the strings?

1. $T_2 = T_1$

2. $T_2 = 2.5 T_1$

3. $T_2 = 0.6 T_1$

4. $T_2 = 0.8 T_1$

5. None of the above

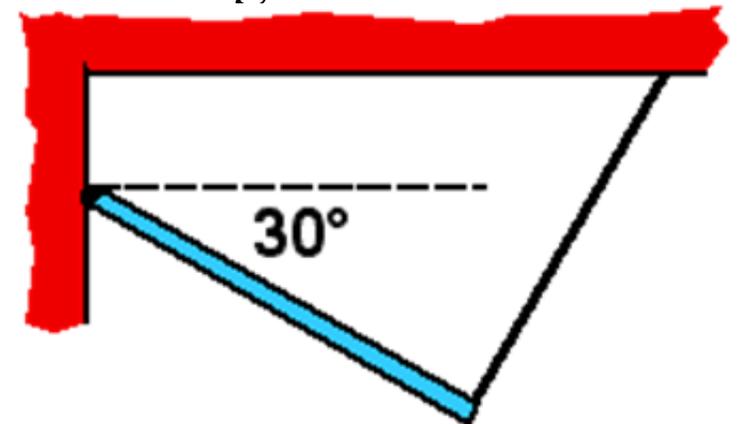
6. Not enough information to determine



Assessing to Learn

A uniform rod is hinged to a wall and held at a 30° angle by a thin string that is attached to the ceiling and makes a 90° angle to rod. Which statement(s) must be true? (At least one of them is true and at least one is false.)

1. The hinge force is purely vertical.
2. The hinge force is purely horizontal.
3. The string tension is equal to the hinge force.
4. The string tension is smaller than the rod's weight.
5. 1 and 3 are true.
6. 2 and 3 are true.
7. 1 and 4 are true.
8. 2 and 4 are true.
9. 3 and 4 are true.
10. Three of the statements are true.



Assessing to Learn

A uniform rod of length $4L$, mass M , is suspended by two thin strings, lengths L and $2L$ as shown. What is the tension in the string at the left end of the rod?

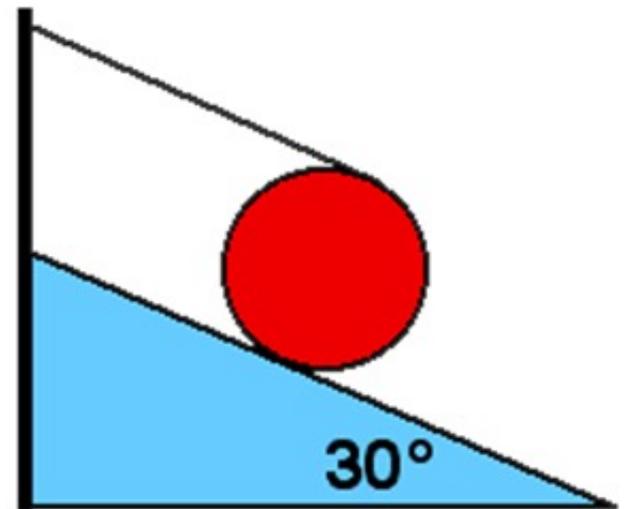
1. Mg
2. $Mg/2$
3. $Mg/3$
4. $Mg/4$
5. None of the above



Assessing to Learn

A uniform disk with mass M and radius R sits at rest on an incline 30° to the horizontal. A string is wound around the disk and attached to the top of the incline as shown. The string is parallel to the incline. What is the tension in the string?

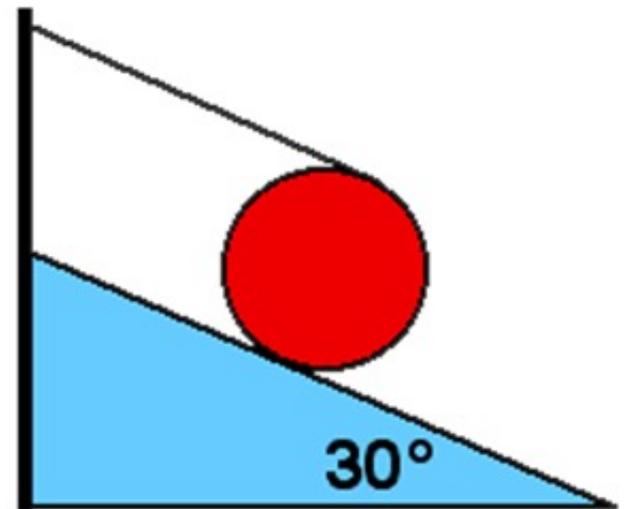
1. Mg
2. $Mg/2$
3. $2Mg/5$
4. $Mg/4$
5. None of the above
6. Cannot be determined



Assessing to Learn

A uniform disk with mass M and radius R sits at rest on an incline 30° to the horizontal. A string is wound around the disk and attached to the top of the incline as shown. The string is parallel to the incline. What is the tension in the string?

1. $Mg/2$, down the incline
2. $Mg/2$, up the incline
3. $Mg/4$, up the incline
4. $Mg/0.86$, down the incline
5. None of the above
6. Cannot be determined



Assessing to Learn

Which situation has the least magnitude of angular momentum about the origin?

A. A 2-kg mass travels along the line $y = 3$ m with speed 1.5 m/s.

B. A 1-kg mass travels in a circle of $r = 4.5$ m about the origin with speed 2 m/s.

C. A disk with $I = 3 \text{ kg}\cdot\text{m}^2$ rotates about its center (on origin) with $\omega = 3 \text{ rad/s}$.

1. A 2. B 3. C 4. Both A and B

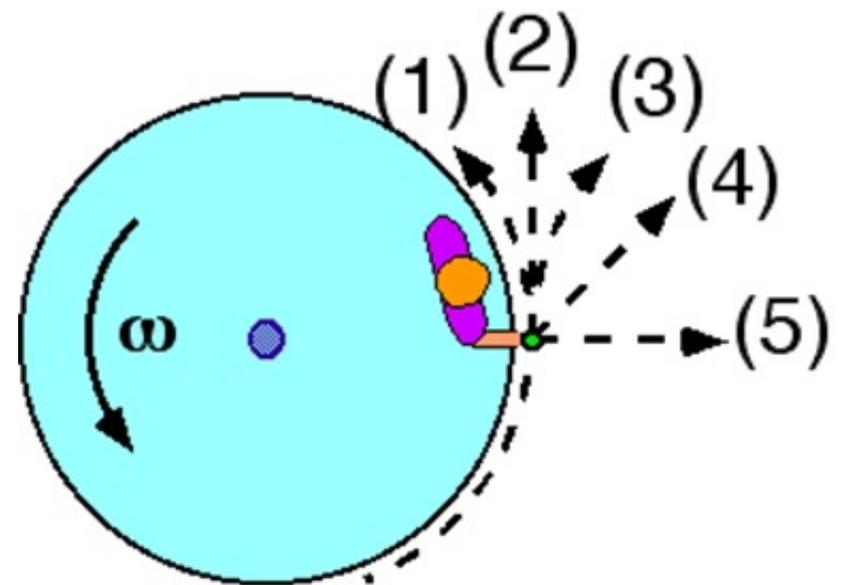
5. Both A and C 6. Both B and C

7. All have the same magnitude angular momentum

Assessing to Learn

A child is standing at the rim of a rotating disk holding a rock. The disk rotates freely without friction. If the rock is dropped at the instant shown, which of the indicated paths most nearly represents the path of the rock as seen from above the disk?

1. path (1)
2. path (2)
3. path (3)
4. path (4)
5. path (5)
6. cannot be determined

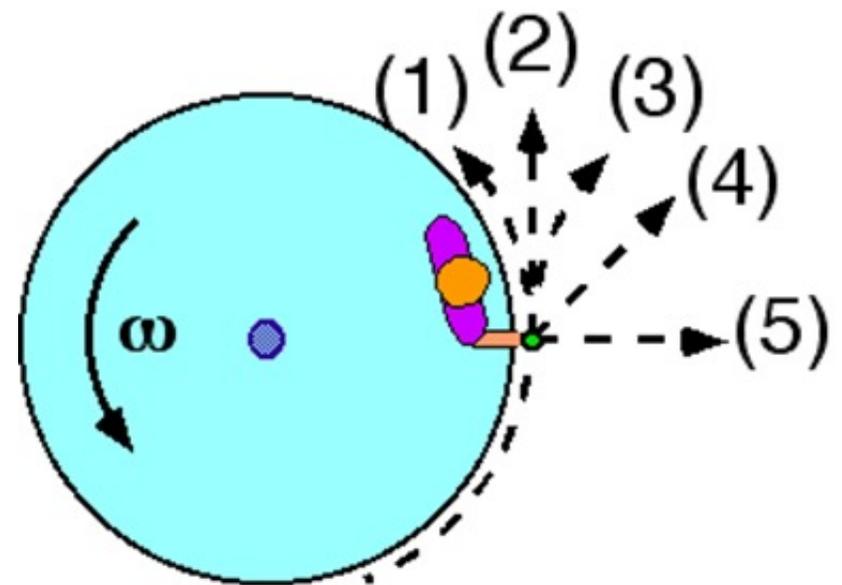


View from above

Assessing to Learn

A child is standing at the rim of a freely rotating disk holding a rock. The disk rotates without friction. The rock is dropped at the instant shown. As a result of dropping the rock, what happens to the angular velocity of the child and disk?

1. increases
2. stays the same
3. decreases
4. cannot be determined

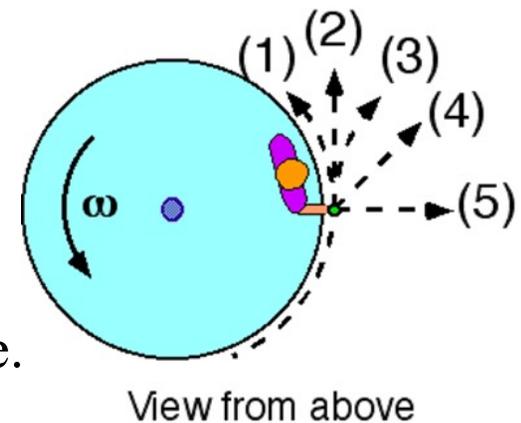


View from above

Assessing to Learn

A child is standing at the rim of a freely rotating disk holding a rock. The disk rotates without friction. The rock is dropped at the instant shown. Which of the following statements is true about the process of dropping the rock?

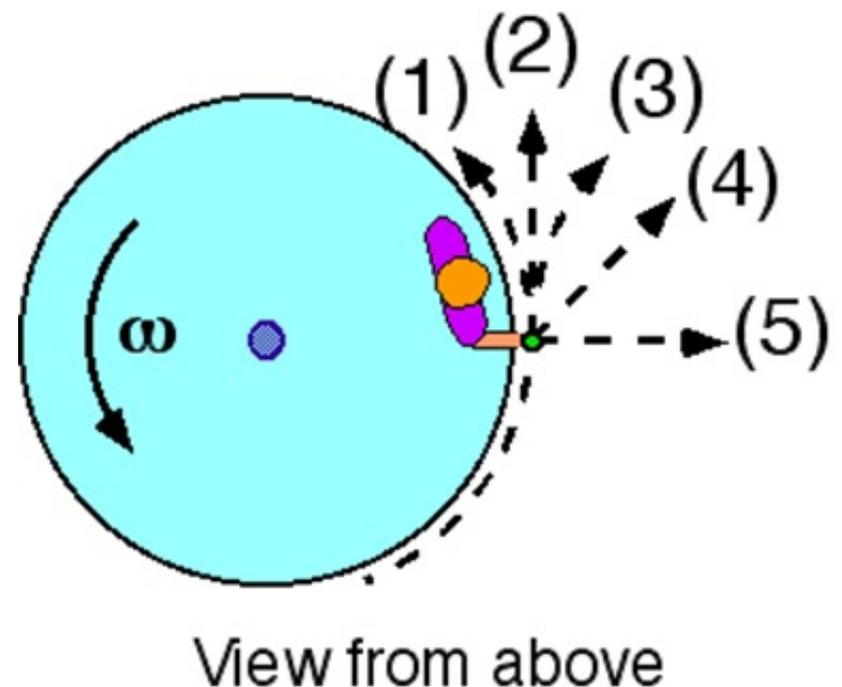
1. Angular momentum is conserved, mechanical energy increases.
2. Angular momentum is conserved, mechanical energy decreases.
3. Angular momentum increases, mechanical energy is conserved.
4. Angular momentum decreases, mechanical energy is conserved.
5. Both angular momentum and mechanical energy are conserved.
6. Both angular momentum and mechanical energy increase.
7. Angular momentum decreases, mechanical energy increases.
8. Angular momentum increases, mechanical energy decreases.
9. Both angular momentum and mechanical energy decrease.
10. The conserved quantities cannot be determined.



Assessing to Learn

A child is standing at the rim of a disk holding a rock. The disk rotates freely without friction. At the instant shown, the child throws the rock *radially outward*. Which of the indicated paths most nearly represents the trajectory of the rock as seen from above?

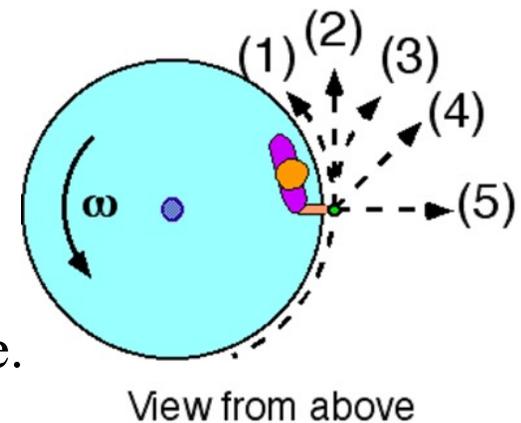
1. path (1)
2. path (2)
3. path (3)
4. path (4)
5. path (5)
6. none of the above
7. cannot be determined



Assessing to Learn

A child is standing at the rim of a rotating disk, and throws a rock *radially outward* at the instant shown. The disk rotates freely without friction. Which of the following statements is correct about the disk-child-rock system as the rock is thrown?

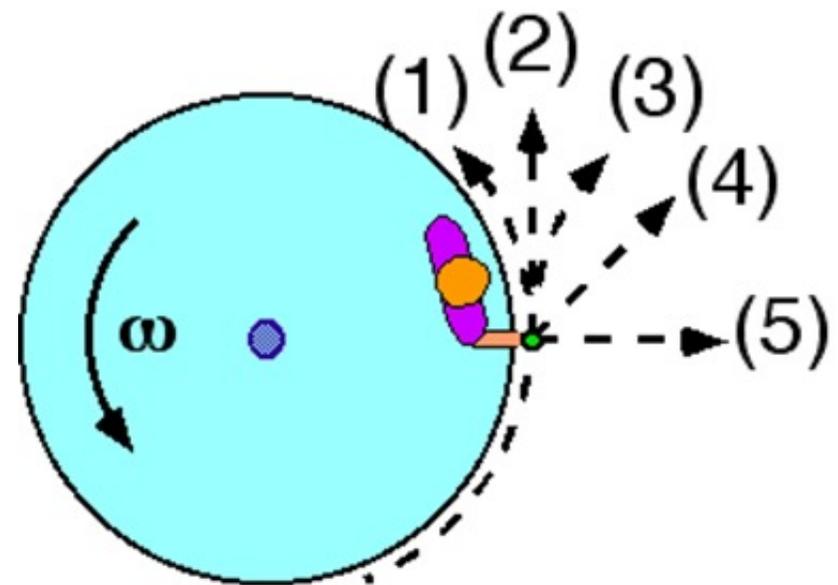
1. Angular momentum is conserved, mechanical energy increases.
2. Angular momentum is conserved, mechanical energy decreases.
3. Angular momentum increases, mechanical energy is conserved.
4. Angular momentum decreases, mechanical energy is conserved.
5. Both angular momentum and mechanical energy are conserved.
6. Both angular momentum and mechanical energy increase.
7. Angular momentum decreases, mechanical energy increases.
8. Angular momentum increases, mechanical energy decreases.
9. Both angular momentum and mechanical energy decrease.
10. The conserved quantities cannot be determined.



Assessing to Learn

A child is standing at the rim of a rotating disk holding a rock, and throws a rock in direction (2) at the instant shown. The disk rotates freely without friction. What happens to the angular speed of the disk?

1. increases
2. remains the same
3. decreases
4. impossible to determine



View from above

Assessing to Learn

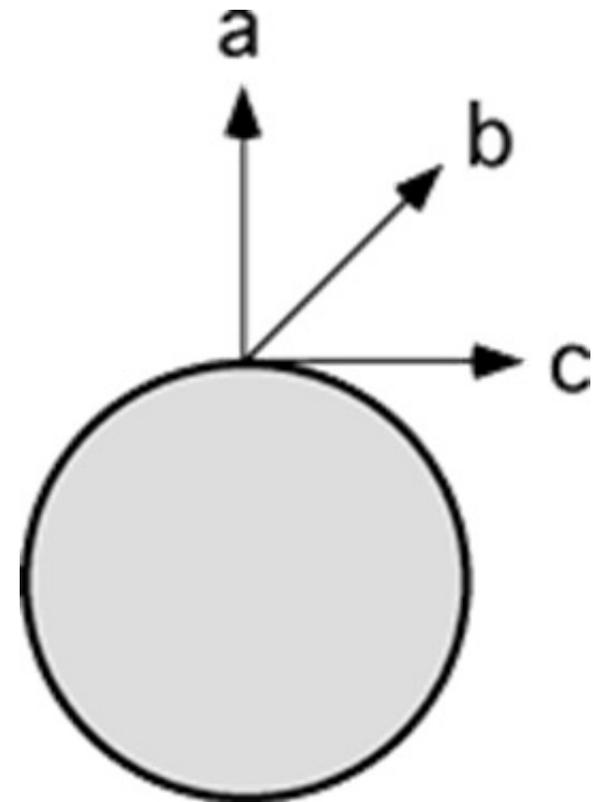
An ice skater begins a spin in the middle of a large rink, but then starts to spin faster by pulling her arms in. Which of the following statements is true?

1. Both kinetic energy and angular momentum are conserved.
2. Kinetic energy is conserved; angular momentum increases.
3. Kinetic energy is conserved; angular momentum decreases.
4. Kinetic energy increases; angular momentum is conserved.
5. Kinetic energy decreases; angular momentum is conserved.
6. Both kinetic energy and angular momentum increase.
7. Kinetic energy increases; angular momentum decreases.
8. Kinetic energy decreases; angular momentum increases.
9. Both kinetic energy and angular momentum decrease.
10. Impossible to determine.

Assessing to Learn

Two masses m_1 and m_2 , having $m_1 > m_2$, are launched with the same speed in the direction a. Which mass reaches the greatest height?

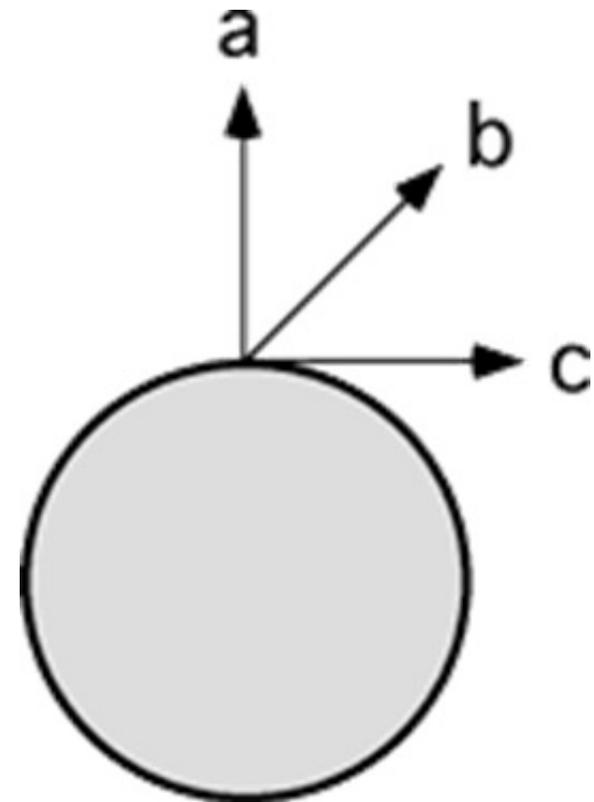
1. m_1
2. m_2
3. Both reach the same height.



Assessing to Learn

Two masses m_1 and m_2 , having $m_1 > m_2$, are launched with the same speed in the direction b. Which mass reaches the greatest height?

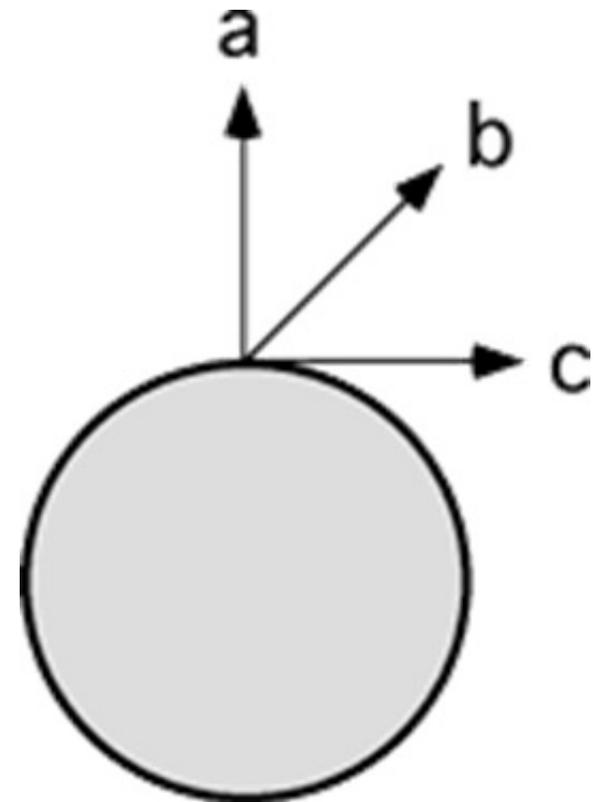
1. m_1
2. m_2
3. Both reach the same height.



Assessing to Learn

A mass m is launched from the surface of the Earth with speed v . The diagram shows three possible launch directions; order them according to the maximum height the projectile will reach, from greatest height to least height.

1. $a = b = c$
2. $a < b = c$
3. $a < b < c$
4. $a < c < b$
5. $a > b = c$
6. $a > b > c$
7. $a > c > b$
8. $b > a > c$
9. $b > a = c$
10. Not enough information.





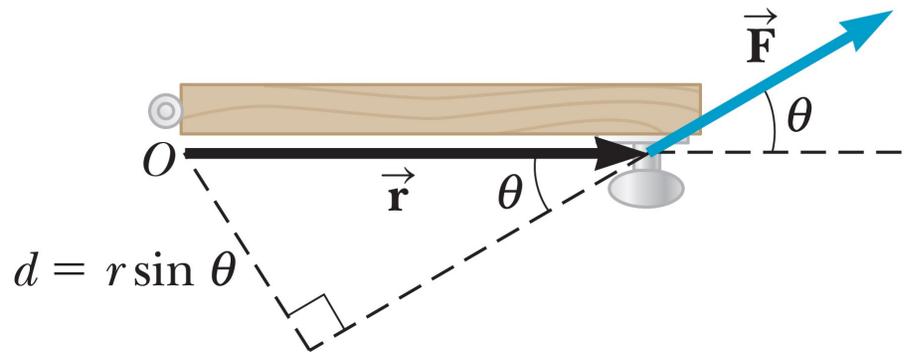
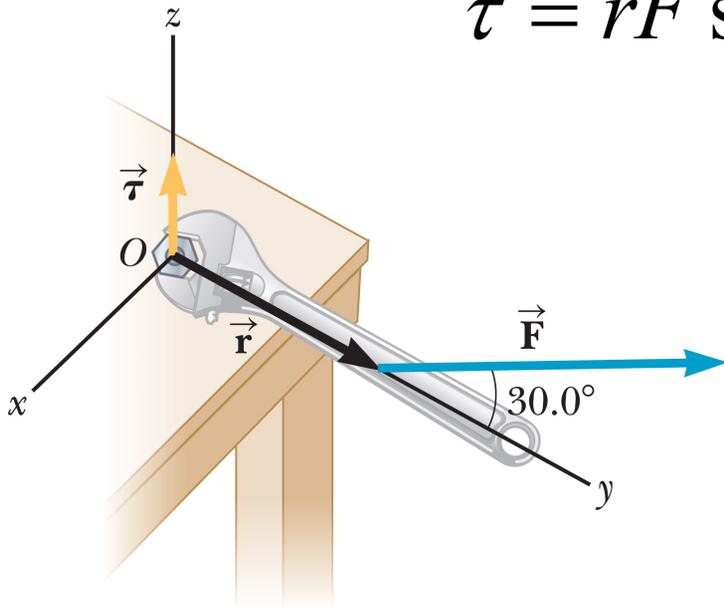
Topic 8: Rotational Equilibrium and Dynamics

TOPIC SUMMARY

Topic Summary

- **Torque**

$$\tau = rF \sin \theta, \quad d = r \sin \theta$$



- **Center of Mass and Its Motion**

$$w = F_g = Mg \rightarrow x_{\text{cg}} = \frac{\sum m_i x_i}{\sum m_i}; \quad y_{\text{cg}} = \frac{\sum m_i y_i}{\sum m_i}; \quad z_{\text{cg}} = \frac{\sum m_i z_i}{\sum m_i}$$

Topic Summary

- **Torque and the Two Conditions for Equilibrium**

- $\Sigma \vec{F} = 0$

- $\Sigma \vec{\tau} = 0$

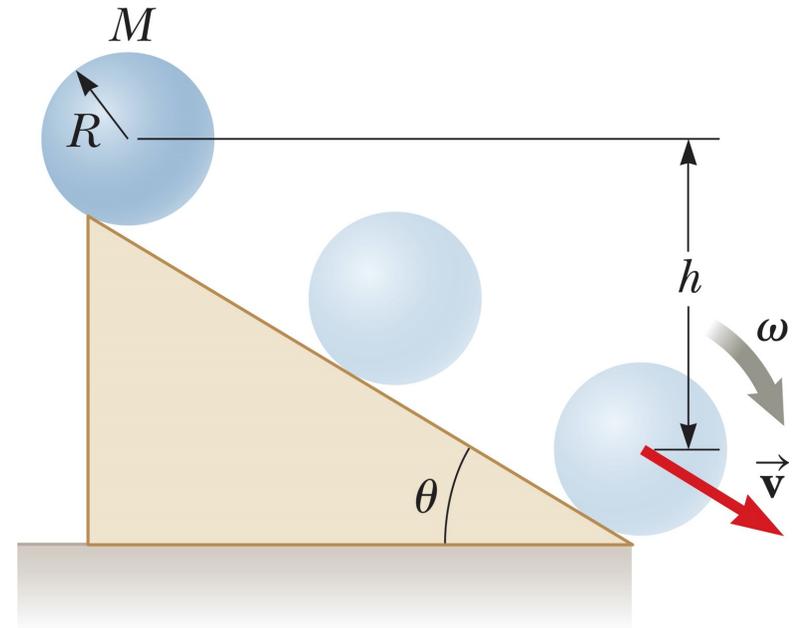
- **The Rotational Second Law of Motion**

$$I \equiv \Sigma mr^2 \quad \Sigma \tau = I\alpha \quad a = r\alpha$$

Topic Summary

- **Rotational Kinetic Energy**

$$KE_r = \frac{1}{2} I \omega^2$$



$$(KE_t + KE_r + PE)_i = (KE_t + KE_r + PE)_f$$

$$W_{nc} = \Delta KE_t + \Delta KE_r + \Delta PE$$

Topic Summary

- **Angular Momentum**

$$L \equiv I\omega$$

$$\Sigma \tau = \frac{\text{change in angular momentum}}{\text{time interval}} = \frac{\Delta L}{\Delta t}$$

$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

**TWO KITTENS ON A SLOPED ROOF. WHICH
ONE SLIDES OFF FIRST?
THE ONE WITH THE LOWEST MEW.**



