HONORS PRE-CALCULUS

Advanced Inequalities Assignment

Name: _____

Date: _____ Period: _____

Instructions: Solve each inequality algebraically over the real numbers. State all final solutions using exact values in **interval notation**. Show all work, including critical numbers and sign analyses (test intervals or factor sign charts).

Part 1: Fractional Exponents & Rational Expressions

1. $f(x) = \frac{3}{7}x^{3/5} + \frac{6}{7}x^{8/5} > 0$

2. $-\frac{4}{9}x^{-4/3} - 8x^{2/3} + \frac{2}{9}x^{-7/3} > 0$

3. $6\sqrt[3]{(x^2)} - \frac{15}{4}x^{-1/3} + \frac{3}{2\sqrt[3]{(x^4)}} \le 0$

4. $f'(x) = \frac{2}{3}x^{-1/4} + \frac{1}{6}x^{3/4} > 0$

5. $\frac{x^{11/3} + x^{5/3}}{\ln(x + 3)} \ge 0$

Part 2: Complex Rational Functions & Derivative Forms

6. $\frac{x^4 - 5x^3 + 6x^2}{x^2 + 2x - 8} < 0$

7. $\frac{3x^2(2x^2 - 9) - 2x(-x^3)}{(9 - x^4)^2} > 0$

8. $\frac{-2x^3(-4x) + 3x^2(25 - x^2)}{(25 - x^2)^3} > 0$

9. $\frac{(4x^2 - 36)(x^2 - 4)^{3/2}}{5} \le 0$

10. $\frac{2x(9 - x^2) - x^2(-2x)}{(9 - x^2)^2} \le 0$

Part 3: Exponential & Logarithmic Inequalities

11. $\frac{4e^x - 9e^{-x}}{2} \le 6$

12. $\frac{e^{2x} + 12e^{-2x}}{4} - 2 > 0$

13. $-6e^{-x} + \frac{x^2}{e^x} < 5xe^{-x}$

14. $3[(e^x)^2 + \frac{1}{e^{-x}}] - 6 > 0$

15. $\frac{(4x^{2x^2/3} - x^{(2x^2+3)/3}) \cdot 3x^e}{3e^{3x} \cdot \ln(3x - 2)} \ge 0$

Part 4: Trigonometric Inequalities

For questions 16–20, solve on the domain $-\pi \le x \le \pi$ unless specified otherwise.

16. $2\cos^4 x - 13\cos^2 x > -4 - 4\cos^2 x$

17. $8\sec x - 14\sec^2 x + 6 \ge 0$

18. $3\cos^4 x + 5 > 21\cos(2x) + 12\sin^2 x - 4$

19. $-20\sec^4 x < -12 + 38\sec^2 x;\ (0 \le x \le \pi)$

20. $$\frac{6\cos x - 6\cos^2 x - 4\sin^2 x\cos x + \sin^2 x\cos^2 x}{\cos^3 x - \frac{1}{3}\cos x} \le 0$$